Managing the Risks of Inflation Expectation De-anchoring*

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Abstract

This paper investigates the implications of potential de-anchoring of medium-term inflation expectations for monetary policy. We propose a monetary policy framework, where the central bank considers de-anchoring risks in a regime switching model. We derive the optimal monetary policy strategy. Optimal monetary policy equates the welfare losses of a more forceful reaction to inflation with the welfare gains of safeguarding credibility. We propose to model de-anchoring risks using a medium-scale regime-switching DSGE model and derive a model-based approach to assess risks of inflation expectation de-anchoring from a real time perspective. We revisit the post-COVID inflation episode and conclude that an explicit looking-through strategy would have raised de-anchoring risks in the euro area to a limited extent.

Keywords: DSGE Estimation, Inflation De-anchoring, Regime Switching

JEL-Codes: D83, D84,E10

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Non-technical summary

When inflation is near its target, central banks often allow for temporary fluctuations of inflation and adjust interest rates only gradually. This approach is common when supply shocks create a trade-off between stabilizing output and inflation, prompting a ”looking through” strategy. However, in times of high inflation or prolonged low inflation, there is a risk that inflation expectations may stray from the central bank's target—a phenomenon known as de-anchoring. In such cases, a stronger policy response is crucial to maintain credibility.

This paper proposes a framework that quantifies the risk of de-anchoring using a regime-switching model. Unlike traditional, purely descriptive indicators from financial markets or surveys, our model-based measure evaluates different scenarios—including forecast baselines and policy counterfactuals—in real-time. This tool helps policymakers understand when a gradual approach is sufficient, such as in environments where inflation is close to target and the stock of credibility is high, and when aggressive action is warranted to prevent expectations from de-anchoring, as in high inflation scenarios with limited stock of credibility.

In an empirical analysis, we analyze long-run inflation expectations according to the participants of the survey of professional forecasters (SPF). We show that long-run inflation expectations are heterogenous, time-varying, and partially driven by current inflation.

We then use a simple economic model to show that a more forceful or aggressive response to inflation deviations from target is optimal if the central bank has limited credibility or inflation expectations become de-anchored. A further increase in forcefulness is optimal if the actions of the central bank influence its credibility. In that case, rate changes do not only affect output and inflation, but also the credibility of the central bank. This calls for a stronger response to inflation deviations from target, once risks to the anchoring of inflation expectations start to

emerge.

Finally, we provide a model-based approach to the risks of de-anchoring. We extend a workhorse macroeconomic model with a regime switching mechanism. In this model, agents attach a higher weight to the de-anchored regime, if the data supports de-anchoring. Using this model we revisit the period since 2010 and identify one period of de-anchoring between 2011 and 2015. The more recent period of the inflation surge is not identified as a de-anchoring event, because of the relatively short-lived increase in inflation and the strong response of the ECB.

Taking into account uncertainty, we construct a measure of risks of de-anchoring, based on stochastic simulations of the de-anchoring model around a baseline. We find that the risks of de-anchoring increase if the central bank is not reacting to a deviation of inflation from target, if the central bank enters into an inflationary or deflationary period with a low stock of credibility, and if the economic environment is characterized by a high degree of uncertainty and volatility.

We furthermore show that if the central bank is 'looking-through' supply side driven inflation, the risks of de-anchoring can increase.

1 Introduction

High inflation is putting central banks to the test. The recent surge in inflation, following an extended period of low inflation and interest rates near the effective lower bound, has radically transformed the policy landscape. A large part of this shift is driven by supply-side factors such as energy prices, output bottlenecks, and economic scarring, which push output and inflation in opposite directions. Unlike demand shocks, these supply shocks present more complex dilemmas for monetary policy, especially in an environment marked by persistent shocks and heightened uncertainty. Since monetary policy cannot directly influence the price of supply factors, a restrictive policy runs the risks of contracting activity severely without reducing inflation effectively. In contrast, an overly accommodative stance risks unmooring inflation and inflation expectations from the announced target. De-anchored inflation expectations can imply self-enforcing dynamics, undermining the credibility of monetary policy.

In this paper, we offer a novel framework for understanding the monetary policy implications of a potential de-anchoring of inflation expectations through a regime-switching approach. We analyse the role of monetary policy to stabilize inflation in an environment where the decisions of the central bank influence the probability of switching between a well-anchored and a de-anchored regime. Our setting assumes that the central bank operates with a constant inflation target, but private agents might doubt that the central bank will bring inflation back to target in the medium term. Instead they rely on a time varying inflation target. By choosing an appropriate policy path, the central bank can gradually bring back inflation to target and realign the private sector's beliefs with the actual inflation target. The more aggressive policy contributes to avoiding a switching to the de-anchored regime or to switch back to the anchored regime. A stronger response to inflation, in order to align expectations, opens a trade-off between the welfare-losses of an overly hawkish stance and the welfare gains associated with an earlier convergence of expectations with the central bank target.

To understand the interplay potential de-anchoring and monetary policy strategy we make the following steps: We describe the optimal monetary policy response to switching credibility in theory. In particular, we derive optimal policy under discretion and under commitment in a simple three equation model where the policy maker influences the probabilities of moving from one regime to another regime. In a further step, we apply the framework to a quantitative policy environment using a macro-economic workhorse model in the euro area. We introduce regime switching and conduct stochastic simulations to derive a measure of risks of de-anchoring of inflation expectations. Finally, we revisit a key historical episode at the peak of inflation to answer the policy question if a looking through strategy would have risked credibility.

We contribute to the literature along three dimensions: First, we extend on optimal policy considerations in regime switching models under constant regime change probabilities to the case where the probabilities are endogenous and driven by the choices of the policy maker. Second, we take the framework of analysis to a quantitative setting in a workhorse DSGE model with regime switching (RSDSGE) to evaluate the euro area and to identify episodes of de-anchoring. Third, we provide a metric of (risks) of inflation de-anchoring in macro models with regime-switching based on a method to conduct stochastic simulations of the RSDSGE model.

With the analysis of optimal monetary policy under regime-switching, respectively forms of heterogeneous expectations and learning we relate to a growing literature. Nakata and Schmidt (2022) analyse optimal monetary policy in a model with zero lower bound on interest rates and sunspot shocks driving the economy into a liquidity trap. We apply their approach in this paper to analyse optimal policy under the constant switching parameter case. Choi and Foerster (2021) analyse simple optimal rules in a regime switching model, finding that it is optimal to switch the parameters of the rule conditional on the prevailing regime. Our approach gives qualitatively similar results, in a simpler, textbook setting: Rather than considering simple optimal rules, we focus on the optimality conditions in a simple three equation model. Our results are confirming that the reaction to inflation and output should be conditional on the regime, and show that the trade-offs are more pronounced if future credibility is considered. Similar to Gasteiger (2021) we find that the central bank needs to act more hawkish if it considers its stock of credibility. Similar to us, Gasteiger (2021) analyses optimal monetary policy in a model with heterogeneous expectation formation processes. He finds that the central bank needs to act more hawkish in comparison to the homogeneous expectation case to stabilize the economy. Overly hawkish policy is however welfare reducing. Lastly, most related to our framework, Adam and Woodford (2012) analyse robust monetary policy allowing private sector expectations to deviate from the model consistent expectations to a small degree. In near rational expectations setting they find a more active policy to be robustly optimal, leading to a more reactive central bank to inflation surprises. Similar to our approach of using a three equation model with the possibility of switching is Gobbi et al. (2019), who impose a functional form for the switching probability and apply their method to the effective lower bound, in contrast we focus on supply shocks driven policy trade-offs.

Our second part provides an empirical perspective on de-anchoring of inflation expectations, based on a workhorse DSGE model, using the methods introduced by Hamilton (1989a). The important milestone for the analysis of non-stationary economic variables with structural changes via a Markov-switching parameter model was introduced by (Hamilton, 1989a). He proposes utilizing regime-switching models to allow for an exogenous, unobservable Markov process to generate shifts between distinct parameter sets that characterize particular regimes. Applying these methods to DSGE models, Davig and Leeper (2007, 2010) show that the minimum state variable solution provides the unique bounded solution to the quasi-linearized model, that can be cast back into a regime-dependent linear model. The solutions and thus the reduced form coefficient matrices account for the transition probabilities of the parameters. Farmer et al. (2009) approach the solution of regime-switching DSGEs using the perturbation solution of the MSV model. Similarily, Maih (2015) uses perturbation methods to solve the regime switching DSGE model and introduces the toolbox RISE. Finally, Foerster et al. (2016) propose to perturb only the set of parameters that influence the stochastic steady-states and thus the ergodic mean of the model. In a recent approach Chang et al. (2019) propose to model an endogenous regime switching via a latent threshold variable. The most frequently used filter for regime-switching applications is the Kim-Nelson filter. Kim (1994) extended the Hamilton filter to regime switching state space models, incorporating regime switching in both the mean and variance. To render regime switching models tractable Kim proposed to approximate the regime specific Gaussian-mixture, i.e. the Kálmán filter posteriors originating from the different regimes, with a single normal distribution. This approximation is of key importance in regime-switching DSGE models. Alternatives, like the interacting multiple model, where the approximation is of the current state instead of the posterior, have been shown to deliver similar filter accuracy, while being slightly more efficient, for details see (Hashimzade et al., 2024). The accuracy is this approximation is of key importance, as if it is inaccurate the forward solution of the DSGE will be arbitrarily off. This approximation is in general accurate if the switching in state variables is small, e.g. if the switching on states does not have an impact on long-run properties, such as asymptotic mean and variance.

In DSGE models, the forward-looking agents base their decisions on the expected future states of the world, and can therefore anticipate regime switches, which in turn can influence the current equilibrium. In our framework the solutions of the two regimes are spanned by the same minimum state space (MSV). This is due to expectations being fundamental (i.e. fully determined by exogenous shocks), as well as some additional properties which allow us to simplify the computations. Note that the only parameters in our model that influence the ergodic mean are the regime transition probabilities. Perturbing them, and taking the first order approximation we can obtain a solution as if each regime was considered in isolation. This simplification is very helpful as it allows to focus on the key object of interest: the regime switching probabilities, i.e. the Markov transition probabilities. This approach allows to analyse those cases where switching is driven by state variables, state shocks and the structure of the underlying model such as the setting of monetary policy (exogenous switching), but not by sunspots (endogenous switching). We highlight that under this definition of exogenous switching, expectations do influence the regime probabilities. Regime probabilities are estimated using a Bayesian updating, and thus are time-varying; furthermore the impact of exogenous shocks is regime-dependent, reflecting differences in expectation formation between regimes. The regime dependent impact of exogenous shock is not only driven by the differences in expectations but is also state dependent. More recently, regime switching models have also been applied to macroeconomic policy analysis. For example, Del Negro et al. (2016) introduced a regime switching DSGE model allowing for changes in monetary policy regimes and showed that the model was able to capture the impact of changes in policy on the economy. Bianchi and Ilut (2017) estimate a DSGE model with switches in the policy mix of fiscal and monetary policy.

1.1 Survey evidence

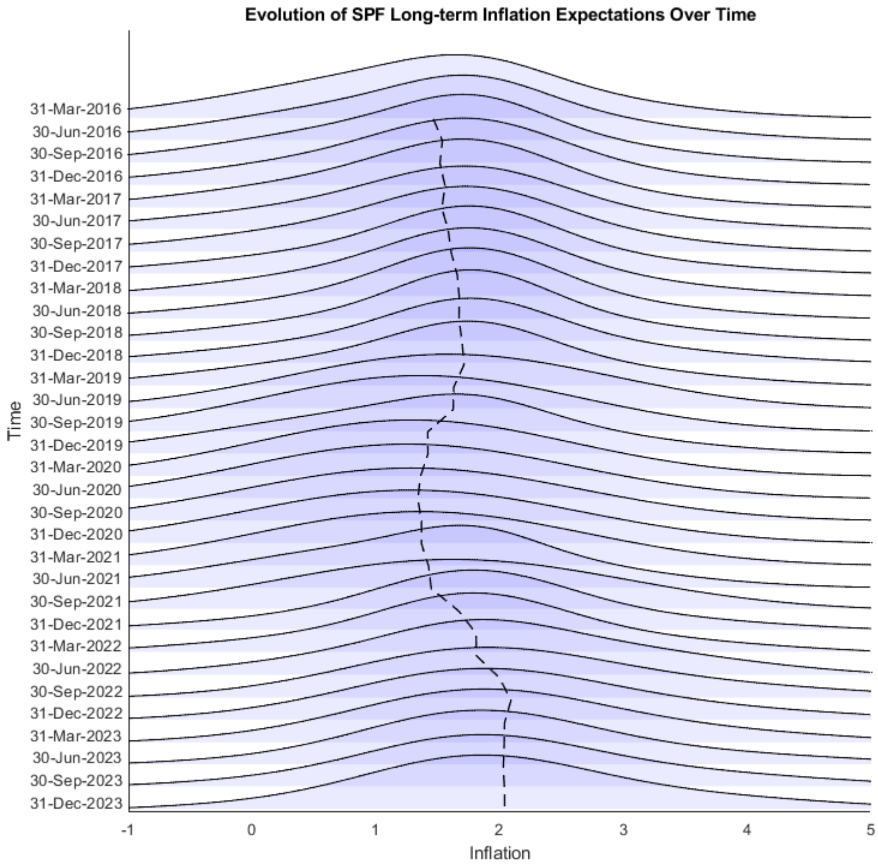
To motivate our approach of modeling the process how expectations might become de-anchored, we first look at evidence on inflation expectations based on surveys. Based on the ECB survey of professional forecasters (SPF) data on euro area inflation expectations and realized inflation, we provide evidence that during the disinflation episode starting in 2011, inflation expectations have shown signs of potential de-anchoring. We model the long-term inflation expectations as a Gaussian-mixture to extract a measure of de-anchored beliefs and estimate a signal extraction problem to calibrate the law of motion for the perceived inflation target under de-anchoring. Corsello et al. (2021) find in their analysis of the SPF that long-term inflation expectations have de-anchored after the 2013-2014 disinflation episode and have become sensitive to surprises in inflation. Dovern et al. (2020) also study the SPF but come to the conclusion that inflation expectations remained anchored, despite responding to realized inflation, and displaying a negative bias in an environment of increased uncertainty.

Figure 1 shows the evolution of long-term SPF expectations over time. It shows that, long-term inflation expectations are time-varying. Both their bias compared to the ECB target, their variance and skewness changes over time, indicating that there are episodes of de-anchoring. These changes follow the trend of inflation, resulting in lower than target expectations until 2021, and higher expectations afterwards. The dispersion of long-term SPF inflation expectations remains broadly unchanged until COVID-19, and starts to increase then, supporting the notion that uncertainty and the rapid increase in inflation both affected long-term inflation expectations. One way to model this pattern is introduce mean-squared learning, reacting both to actual inflation and to uncertainty. We also note that inflation expectations did not change abruptly, but show some variations around the announced target. These variations range from values around 1.5% between late 2019 and end 2021, to values slightly above 2 % during the inflation surge.

Appendix (F) establishes that there is a correlation between observed inflation and the mean of pooled expected long-term inflation. This can be interpreted as time-varying de-anchored portion of beliefs. Here we focus on the modelling of de-anchored expectations. We model SPF expectations using a Gaussian-mixture model, that is, we break down the survey data into two

Figure 1: Long-term Inflation Expectations in the ECB Survey of Professional Forecasters

The chart shows that the evolution of the long-term SPF inflation expectations. Changes in the distributions affects both the mean (dashed line) and the variance (dispersion of the densities) and the skewness.



Notes: The chart shows the probability density of the SPF long-term inflation expectations. The cross sectional pooled probability distribution of the long-term inflation are fitted kernel distribution. *Sources:* SPF, Authors' calculations. Sample: 2002 June - 2023 December.

main groups. One group that is centered at the ECB's target and the other that is de-anchored. Let us decompose the long-term inflation expectations distributions from the SPF into a mix of these two groups at any time:

$$D_t = p * \mathcal{N}(\mu_1, \sigma_1^2) + (1-p) * \mathcal{N}(\mu_2, \sigma^2) \quad (1)$$

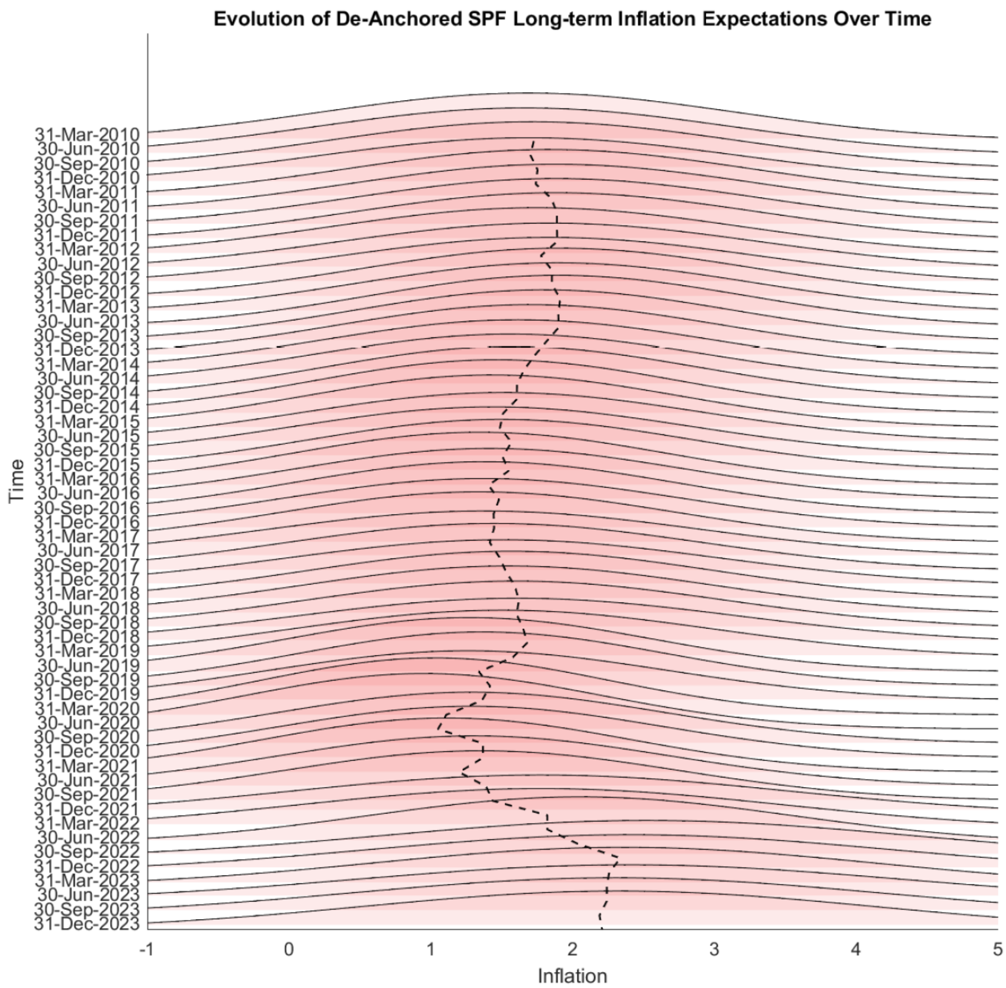
Where D_t is the pooled distribution of long-term inflation expectations according to the SPF at a given time t.

We assume that the first group takes the European Central Bank target as given. The second group assumes that the target is time-varying and unobserved, implying a higher uncertainty.

We use an Expectation Maximization algorithm to figure out the parmaeters of the components of the Gaussian-mixture representing the SPF expectations, according to equation 1. We also account for changes in the European Central Bank's target, by imposing a 1.9 percent mean prior to 2021 and a 2.0 percent mean since 2021 for the first group. Figure 2 shows the de-anchored component of the long-term SPF inflation expectations. It shows that there is a drift in the inflation expectations below the ECB's target over time starting in 2014 and culminating in a de-anchoring to the downside in late 2019 and in the run-up to the COVID-19 pandemic. Following the post-COVID surge in inflation, the de-anchored component's mean and variance increased as well, shifting expectations to the upside of the new symmetric inflation target.

Figure 2: De-Anchored Component of Long-term Inflation Expectations in the ECB Survey of Professional Forecasters

The chart shows that the evolution of the de-anchored component of the Gaussian-mixture model for the long-term SPF inflation expectations.



Notes: The chart shows the probability density of the de-anchored component of the SPF long-term inflation expectations. The cross sectional pooled probability distribution of the long-term inflation are fitted with a Gaussian-kernel distribution. *Sources:* SPF, Authors' calculations. Sample: 2002 June - 2023 December.

Having established that inflation expectations are heterogeneous and time-varying and can be approximated by a Gaussion mixture, in the next step, we provide an empirical calibration to the time-varying inflation target process. For the empirical specification we model imperfect credibility following Erceg and Levin (2003) and Bordo et al. (2017). We assume SPF participants face a simple signal extraction problem and try to infer the changes in inflation target and monetary policy shocks from a noisy signal. To understand it, consider the modified Taylor rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[(r^* + \pi_t) + \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y(\hat{y}_t)] + e_t^{MP} \tag{2}$$

Where i_t is the short-term policy interest rate, π_t is annual HICP inflation, π_t^* is the long-run inflation target percieved, $\hat{y}_t$ is the output gap GDP, and e_t^{MP} denotes the monetary policy shock to the policy reaction function.

SPF participants cannot directly observe neither the long-run inflation target nor the monetary shock ε_t^{MP}; but observe economic conditions. From this they can infer a composite shock $\phi_t = -\pi_t^* + e_t^{MP}$, which is a combination of the inflation target shock and the monetary policy shock. The perceptions of the unobserved components can then be cast into a state space to follow a first-order vector autoregression:

$$\begin{bmatrix} \pi_t^* \\ e_t^{MP} \end{bmatrix} = \begin{bmatrix} c_{\pi_t^*} \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_{\pi^*} & 0 \\ 0 & \rho_{\varepsilon^{MP}} \end{bmatrix} \begin{bmatrix} \pi_{t-1}^* \\ e_{t-1}^{MP} \end{bmatrix} + \begin{bmatrix} \sigma_{\pi^*} & 0 \\ 0 & \sigma_{\varepsilon^{MP}} \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi^*,t} \\ \varepsilon_{MP,t} \end{bmatrix} \tag{3}$$

Following Bordo et al. (2017) we estimate the signal extraction problem of the perceived inflation target for the de-anchored expectations by assuming that monetary policy shocks are white noise, and the exogenous innovations to inflation target shocks and monetary policy surprises are uncorrelated. We take the mean of de-anchored component from Figure 2 as the estimate for the de-anchored inflation target, and estimate the co-efficient of the unobserved state space model on the difference of the perceived inflation and the ECB's inflation target, i.e. inflation target gap by setting $c_{\pi_t^*} = \pi^{*,ECB}$.

The OLS estimates for the resulting AR(1) model for the inflation target gap is : The results of the signal extraction model, shown in Table 1, reveals the key insights regarding the dynamics of the perceived inflation target. The constant term is estimated to be zero indicating

Table 1: signal Extraction Problem of the Perceived Inflation Target
Coefficients De-Anchored Component of Long-term SPF Inflation Expectations

	Value	**Standard Error**	**T Statistic**	**P Value**
Constant	0.002	0.011	0.147	0.883
AR(1)	0.953	0.038	25.341	1.1385e-141
Variance	0.005	0.001	9.062	1.2843e-19

that the percieved target is not biased in sample. The autoregressive parameter AR(1) is estimated at 0.953, demonstrating a strong persistence in the inflation target perceptions, which is significant at an exceedingly low p-value, suggesting that past values of the perceived inflation target heavily influence current perceptions.[1] These findings highlight that imperfect credibility in monetary policy is highly persistent and driven by past deviations of inflation from target.

In line with this evidence we define our structural model definition of de-anchoring as an episode where inflation deviates persistently from target and is gradually shifting the perceptions of the economic agents towards a notion of limited credibility of the central bank. Under this definition of limited credibility, agents might find it preferable to deviate from the announced inflation target when the central bank fails to deliver on it's price stability commitment.

The possibility of inflation expectations becoming de-anchored has implications for the formulation of the monetary policy strategy of central banks targeting a specific inflation rate. In the next part, in an optimal monetary policy exercise, we derive how central banks facing a potential de-anchoring shall conduct policies. We show that under de-anchoring risks the central bank should not only care about inflation realizations but also the medium-term inflation expectations of households and firms. This additional consideration is calling for a more forceful reaction of the central bank to deviations of inflation from the announced target.

2 Optimal monetary policy

To analyse optimal monetary policy, we take the 3-equation NK DSGE model and augment it with a Markov switching process. The resulting model is then composed of three key economic equations: the Phillips curve, the New Keynesian aggregate demand (IS) curve, the monetary

[1]We take these results as indication for the modelling of the time-varying inflation target in the theoretical model, but do not follow the exact calibration, because of differences of the empirical model shown here and the economical model used later on.

policy rule, and the Markov-switching process.

The New Keynesian Phillips curve (4) relates the inflation gap to the output gap and to expected inflation. The IS curve (5) describes how output responds to changes in the real interest rate, constructed as the nominal interest rate adjusted for expected inflation.

These two equations are typically written as:

$$\widehat{\pi}_t = \kappa \widehat{y}_t + \beta E_t[\widehat{\pi}_{t+1}] + u_t \tag{4}$$

$$\widehat{y}_t = E_t[\widehat{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\widehat{\pi}_{t+1}] - r_t^n) + g_t \tag{5}$$

where u_t is a supply shock and g_t is a demand shock

We express inflation as the inflation gap $\widehat{\pi}_t = (\pi_t - \pi_t^*)$, where π_t is the inflation rate between periods $t-1$ and t and π_t^* is the inflation target that might vary over time or be subject to misconceptions. We assume that the central bank has a constant inflation target $\pi^{*,CB}$ but the perceived target of households differs between the high (h) and low (ℓ) credibility states. In the high-credibility regime, the perceived target of households is identical to the central bank's target but deviates from it in the low-credibility regime.[2] The output gap is denoted by $\widehat{y}_t$, i_t is the level of the nominal interest rate between periods t and $t+1$, and r_t^n is the exogenous natural real rate of interest. E_t is the rational expectation operator conditional on information available in period t. The parameter $\beta \in (0,1)$ denotes the subjective discount factor of the representative household, $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, and κ represents the slope of the New Keynesian Phillips curve.

Switching probabilities

The perceived inflation target follows a two-state Markov process. In particular, $\pi^{*,HH,i}$, for $i = h, \ell$, takes the value of either $\pi^{*,HH,h} = \pi^{*,CB}$ in the high credibility state, or $\pi^{*,HH,\ell} \neq \pi^{*,CB}$ in the low credibility state. Furthermore we define the gap between the agents (perceived) inflation target and the actual inflation target of the central bank as $k^i = \pi^{*,HH,i} - \pi^{*,CB}$.

[2]At this point we do not make a assumption on the sign of the deviation. In the following, we differentiate between downward and upward deviations.

The transition probabilities are assumed to be constant.[3]

$$\text{Prob}(\pi_{t+1}^{*,HH,h} = \pi^{*,HH,h} | \pi_t^{*,HH,h} = \pi^{*,HH,h}) = p^h \tag{6}$$

$$\text{Prob}(\pi_{t+1}^{*,HH,\ell} = \pi^{*,HH,\ell} | \pi_t^{*,HH,\ell} = \pi^{*,HH,\ell}) = p^\ell \tag{7}$$

p^h is the probability of staying in the high state in the next period when the economy is in the high state today. p^ℓ is the probability of staying in the low state when the economy is in the low state today. The switching probabilities are the respective complementary probabilities $1 - p^H$ and $1 - p^\ell$.

Society's objective and the central bank's problem

Society's welfare, at time t is given by the expected discounted sum of future utility flows,

$$V_t = u(\widehat{\pi}_t^{HH,i}, \widehat{y}_t) + \beta E_t V_{t+1} \tag{8}$$

where the inflation gap is defined as $\widehat{\pi}_t^{HH,i} = (\pi_t - \pi_t^{*,HH,i})$ and the contemporaneous utility function, $u(\cdot,\cdot)$, is assumed to be given by the standard quadratic function of the inflation gap and the output gap:

$$u(\widehat{\pi}, \widehat{y}) = -\frac{1}{2}\left((\pi - \pi^{*,HH})^2 + \lambda \widehat{y}^2\right). \tag{9}$$

The objective function can be motivated by a second-order approximation to the household's preferences. In such a case, λ is a function of the structural parameters and is given by $\lambda = \kappa/\theta$.

Monetary policy is delegated to a central bank. The value for the central bank is given by

$$V_t^{CB}(\widehat{\pi}^{CB}, \widehat{y}) = u^{CB}(\widehat{\pi}^{CB}, \widehat{y}) + \beta E_t[V^{CB}(\widehat{\pi}^{CB}, \widehat{y})] \tag{10}$$

where the central bank's contemporaneous utility function, $u^{CB}(\cdot,\cdot)$, is given by

$$u^{CB}(\widehat{\pi}^{CB}, \widehat{y}) = -\frac{1}{2}\left((\pi - \pi^{*,CB})^2 + \lambda \widehat{y}^2\right). \tag{11}$$

[3]In section 2.2 this assumption is relaxed by allowing endogenous probabilities of regime switching.

The value function of the central bank differs from the society's value function only with respect to the definition of the inflation gap.

A Markov-Perfect equilibrium is defined as a set of time-invariant value and policy functions $\{V^{CB}(\cdot), y(\cdot), \pi(\cdot), i(\cdot)\}$ that solves the central bank's problem above, together with society's value function $V(\cdot)$, which is consistent with $y(\cdot)$ and $\pi(\cdot)$.

2.1 Optimal policy under constant switching probabilities

We start by formulating the central bank's problem. The central bank's objective is thus to minimize its loss function (10) subject to the Phillips curve (4), aggregate demand curve (5), and a Markov-switching process for the inflation target.

For the case of non-zero switching probabilities Choi and Foerster (2021) propose to derive simple, optimal rules. We follow the approach by Nakata and Schmidt (2022) defining the system of linear equations:

Definition 1. *The regime dependent optimal monetary policy equilibrium is defined as a vector* $\{y_t^h, \pi_t^h, i_t^h, y_t^\ell, \pi_t^\ell, i_t^\ell\}$ *that solves the following system of linear equations:*[4]

$$\widehat{y}_t^h = [p^h \widehat{y}_{t+1}^h + (1-p^h)\widehat{y}_{t+1}^\ell] + \frac{1}{\sigma}[p^h \widehat{\pi}_{t+1}^h + (1-p^h)\widehat{\pi}_{t+1}^\ell - i_t^h + r_t^n], \tag{12}$$

$$\widehat{\pi}_t^h = \kappa \widehat{y}_t^h + \beta[p^h \widehat{\pi}_{t+1}^h + (1-p^h)\widehat{\pi}_{t+1}^\ell], \tag{13}$$

$$0 = \kappa \widehat{\pi}_t^h + \lambda \widehat{y}_t^h, \tag{14}$$

$$\widehat{y}_t^\ell = [p^\ell \widehat{y}_{t+1}^\ell + (1-p^\ell)\widehat{y}_{t+1}^h] + \frac{1}{\sigma}[p^\ell \widehat{\pi}_{t+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+1}^h - i_t^\ell + r_t^n], \tag{15}$$

$$\widehat{\pi}_t^\ell = \kappa \widehat{y}_t^\ell + \beta[p^\ell \widehat{\pi}_{t+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+1}^h], \tag{16}$$

$$0 = \kappa(\widehat{\pi}_t^\ell + \pi_t^{*,\ell} - \pi_t^{*,CB}) + \lambda \widehat{y}_t^\ell, \tag{17}$$

Setting the shadow prices zero we get the usual recursion that

The system defined in equations (12) to (17) shows that the optimality conditions follow

[4] Please note that the notation is changing at this point. While the formulation of the objective of the households and the central bank is defined from the viewpoint of the respective decision maker, for the following analysis it is more useful to differentiate between the high credibility regime (h) and the low credibility regime (ℓ).

the usual logic: interest rates are set to balance the output and inflation gaps, such $\hat{\pi}_t = -\frac{\lambda}{\kappa}\hat{y}_t$ holds in each period Clarida et al. (1999). Our results deviate from these results in terms of the expectations of the continuation values in each regime are weighted with the probability of switching to or staying in the respective regime.

Case of discretion

In a first step, assume that the central bank does not have a commitment technology and start by looking at the two regimes separately to provide an intuition for the optimal monetary policy implications of the regime switching model. If the economy is in the high credibility regime and the probability of switching to the low credibility regime is zero ($p^h = 1$), the optimal policy prescription follows the textbook prescription as in Woodford (2004) or Clarida et al. (1999). The optimal policy can then be characterized by setting inflation proportional to the output gap. In the simple model output gap stabilisation and inflation stabilisation coincide:[5]

$$\widehat{\pi}_t = -\frac{\lambda}{\kappa}\widehat{y}_t \tag{18}$$

We can combine this condition with (4) and (5) to come up with an optimal interest rate equation of the following form:

$$i_t^h = \sigma E_t[\widehat{y}_{t+1}^h] + \left[1 + \frac{\kappa\beta\sigma}{\kappa^2+\lambda}\right] E[\widehat{\pi}_{t+1}^{HH,h}] + \frac{\sigma\kappa}{\kappa^2+\lambda} u_t + \sigma g_t \tag{19}$$

If the economy is in the low credibility regime and the probability of switching back to the high credibility environment is zero ($p^\ell = 1$), the perceived inflation target of the agents differs from the inflation target of the central bank, $k^\ell = \pi^{*,\ell} - \pi^{*,CB} \neq 0$. The central bank cannot influence the perceived inflation target of the agents or the switching probability per-se and is confronted with $k^\ell \neq 0$. This implies a discrepancy between the socially optimal solution to equation (8) and the central bank objective according to equation (10), because the former is defined with respect to the perceived inflation gap, while the latter is based on the central bank inflation gap.

[5] Blanchard and Gali (2007) coin this relation as the 'divine coincidence', holding in simple models for the optimal policy reaction to demand shocks.

The implied change in the optimal rule can be written as an additional term in the target gap (k).

$$i_t^\ell = \sigma E_t[\widehat{y}_{t+1}^\ell] + \left[1 + \frac{\kappa\beta\sigma}{\kappa^2+\lambda}\right] E[\widehat{\pi}_{t+1}^{HH,\ell}] + \frac{\sigma\kappa}{\kappa^2+\lambda}u_t + \frac{\sigma\kappa}{\kappa^2+\lambda}k^\ell + \sigma g_t \tag{20}$$

Note that equation (20) is expressed in terms of the perceived inflation gap of the households. For reasonable economic calibrations the coefficient of k^ℓ is positive. In case of a positive gap (perceived target being higher than the central bank's target, $k^\ell > 0$), the rate from the perspective of the household is higher (by the factor $\frac{\sigma\kappa}{\kappa^2+\lambda}k$) than the rate under full credibility (19).

We can also express relation (20) in terms of the actual target gap of the central bank:

$$i_t^\ell = \sigma E_t[\widehat{y}_{t+1}^\ell] + \left[1 + \frac{\kappa\beta\sigma}{\kappa^2+\lambda}\right] E[\widehat{\pi}_{t+1}^{CB}] + \frac{\sigma\kappa}{\kappa^2+\lambda}u_t + \frac{(1-\beta)\sigma\kappa - 1}{\kappa^2+\lambda}k^\ell + \sigma g_t \tag{21}$$

For $((1-\beta)\sigma\kappa < 1)$ the coefficient of k^ℓ is negative.[6] For $k^\ell > 0$, the rate is lower than the rate the central would choose if households agree with the official target. The bias in the target is opening a trade-off for the central bank between closing the actual inflation gap or the perceived inflation gap of households. The bias can be reduced by putting a higher weight λ on output in the loss function of the central bank.[7] For $\lambda \to \infty$, the bias disappears but implies that the central bank is focusing solely on a closed output gap. From that perspective, the limited credibility would result in an optimal policy with further loss of credibility and inflation driven by inflationary expectations shocks. The central bank could also eliminate the bias by adjusting the target of the central bank to the perceived target of the households, with similar repercussions on its credibility.

Optimal policy under discretion implies that the expectational terms are not considered in the optimization. The optimal policy choice is then conditional on the current state of the economy and switches with the regimes.[8] The central bank will set rates according to equation

[6]This expression is negative for most economically plausible parameter combination.

[7]Note that the bias in our approach can be reduced by setting a higher weight on output deviations, while Clarida et al. (1999) for the case of a bias in the targeted output gap of the central bank find a higher weight on inflation to be welfare improving.

[8]This result is in line with the analysis of Choi and Foerster (2021) based on simple optimal rules, who finds switching rules to be optimal

(19) when the economy is in the high credibility regime and set rates according to equation (20) once the economy switches to the low credibility regime.

Case of commitment

We assume that the central bank has a commitment technology for setting interest rates but lacks a commitment mechanism for the inflation target itself. Under commitment, the central bank's optimality condition differs from discretion by incorporating intertemporal considerations, as reflected in the standard model's result that can be written as:[9]

$$(\widehat{y}_{t+i} - \widehat{y}_{t+i-1}) = -\frac{\kappa}{\lambda}\widehat{\pi}_{t+i} \tag{22}$$

This condition reflects that any deviation from the static inflation-output gap trade-off must be offset by a credible pledge to future deviations. Unlike under discretion, where the policy rate is determined period-by-period based on contemporaneous conditions, commitment ensures that expectations incorporate the dynamic trade-off, making policy forward-looking.

To make the derivation tractable, we assume that expectations are the probability-weighted average of the unique rational expectations solutions as in Davig and Leeper (2007) and Farmer et al. (2009, 2011). This assumptions renders the regime-switching model to be represented as a single model with expanded states accounting for the regimes. We acknowledge that this solution is just one of the multiple approaches of dealing with the regime-switching forward solution that on the other hand enables close-to-closed form analytic treatment of the problem. For an alternative approach of optimal policy in a Markov-switching rational expectations models see Blake and Zampolli (2011).

Appendix C derives the optimal Taylor-type rule under commitment under constant switching probabilities given the assumptions. The resulting interest rate rule under commitment in the high credibility state takes the form:

[9] Assuming that the initial conditions satisfy the optimality criterion under discretion: $\widehat{\pi}_0 = -\frac{\lambda}{\kappa}\widehat{y}_0$ as in Clarida et al. (1999).

$$
\begin{aligned}
i_t^h &= r_t^n + \pi^{*,CB} + (1-p^h)k \\
&+ \left[p^h - \frac{\beta\kappa\sigma}{\lambda}A\right] E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right] \\
&+ \left[(1-p^h) - \frac{\beta\kappa\sigma}{\lambda}B\right] E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right] \\
&+ \frac{\beta\kappa\sigma}{\lambda}\Bigg\{ p^h \underbrace{\left(E_t\left[\widehat{\pi}_{t+2}^{HH,h}\right] - E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right]\right)}_{\Delta\widehat{\pi}_{t+1}^{HH,h}} + (1-p^h)\underbrace{\left(E_t\left[\widehat{\pi}_{t+2}^{LL,\ell}\right] - E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right]\right)}_{\Delta\widehat{\pi}_{t+1}^{LL,\ell}} \\
&\quad + C\underbrace{\left(E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right] - E_t\left[\widehat{\pi}_{t}^{HH,h}\right]\right)}_{\Delta\widehat{\pi}_{t}^{HH,h}} + D\underbrace{\left(E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right] - E_t\left[\widehat{\pi}_{t}^{LL,\ell}\right]\right)}_{\Delta\widehat{\pi}_{t+1}^{LL,\ell}}\Bigg\} + \sigma g_t
\end{aligned}
\tag{23}
$$

With the composite coefficients A, B, C, D defined in Appendix C.

Timeless commitment translates into a Taylor-type rule that aims to stabilize the growth of the inflation gap. Note that if there is no regime switching, and $p^h = 1$, and agents expect to remain in the high-credibility regime with certainty, the policy rule simplifies to the case of commitment in Clarida et al. (1999). In this case, the inflation target gap k of the low credibility regime plays no role, as credibility is fully intact, and agents align their expectations with the central bank's target and the drift in the Taylor-type rule becomes the neutral rate and the inflation target of the central bank. However, when $p^h < 1$, meaning credibility is at risk, the term $(1-p^h)k$ enters the Taylor-type rule as a drift, creating a bias even under the credible regime's commitment compared to single regime baseline. This deviation accounts for the future possibility that the perceived and actual targets diverge. The coefficients in front of one- and two-periods ahead inflation expectation terms reflects the central bank's need to respond forcefully to inflation expectations more than one-to-one, obeying the Taylor principle.

The interplay between p^h and k reveals an important policy trade-off: if the exogenous probability of losing credibility is high, that is credibility is weak and households perceive a different inflation target, then the central bank must accommodate this perception. As usual, a higher weight on output stabilization λ reduces the sensitivity of the policy rate to inflation deviations.

A similar expression holds in the low regime, with the regime–specific indices ℓ switched h where appropriate.

2.2 Time-varying, endogenous probability of switching

Now we consider the case where the decisions of the policymakers influence the probability of staying in the regime or switching to the other regime, i.e. $\frac{\partial p_t^{h,\ell}}{\partial i_t} \neq 0$. We are analyzing time-varying, endogenous Markov-switching properties.

This implies the Markov-switching transition matrix:

$$\Pi_t = \begin{bmatrix} p_t^h & 1-p_t^h \\ 1-p_t^\ell & p_t^\ell \end{bmatrix} \tag{24}$$

We can rewrite $p_t^h = p^h(1-\Delta_t^h)$ and $p_t^\ell = p^\ell(1-\Delta_t^\ell)$, to express the optimality condition as a perturbation around the constant probability solution.

Then the envelope condition of the value function of the central bank's optimality condition can be cast into the following format, where $V^{CB,TVP}$ denotes the value function under a time-varying switching probability and $V^{CB,CP}$ the value function under constant switching probabilities: The first order condition of the value function (10)

$$-\frac{\partial u_t^{CB,TVP}}{\partial i_t} = \beta \frac{\partial E_t V_{t+1}^{CB,TVP}}{\partial i_t} \tag{25}$$

can be decomposed according to the regime switches:

$$-\frac{\partial u_t^{CB,TVP}(p_t^h)}{\partial i_t} = \frac{\partial(\beta p_t^h V_{t+1}^h + \beta(1-p_t^h)V_{t+1}^\ell)}{\partial i_t} \tag{26}$$

Using the conjecture that the solution is linear in the states, the chain rule implies that:

$$\frac{\partial u_t^{CB}\left(p^h(1-\Delta_t^h)\right)}{\partial i_t} = \frac{\partial u_t^{CB}(p^h)}{\partial i_t} - u_t^{CB}(p^h)\frac{\partial \Delta_t^h}{\partial i_t}. \tag{27}$$

Furthermore we can also write that:

$$\frac{\partial(\beta p_t^h V_{t+1}^h + \beta(1-p_t^h)V_{t+1}^\ell)}{\partial i_t} = \frac{\partial(\beta p^h V_{t+1}^h + \beta(1-p^h)V_{t+1}^\ell - \beta\Delta_t^h V_{t+1}^h + \beta\Delta_t^h V_{t+1}^\ell)}{\partial i_t} =$$
$$\beta p^h \frac{\partial(V_{t+1}^h)}{\partial i_t} + \beta(1-p^h)\frac{\partial(V_{t+1}^\ell)}{\partial i_t} - \beta\frac{\partial(\Delta_t^h(V_{t+1}^h - V_{t+1}^\ell))}{\partial i_t} =$$
$$\beta p^h \frac{\partial(V_{t+1}^h)}{\partial i_t} + \beta(1-p^h)\frac{\partial(V_{t+1}^\ell)}{\partial i_t} - \beta p^h(V_{t+1}^h - V_{t+1}^\ell)\frac{\partial\Delta_t^h}{\partial i_t} - \beta p^h\Delta_t^h\frac{\partial(V_{t+1}^h - V_{t+1}^\ell)}{\partial i_t}. \tag{28}$$

Collecting the terms we find that:

$$\underbrace{-\frac{\partial u_t^{CB,CP}}{\partial i_t}}_{\text{Constant probability case}} + u_t^{CB}\frac{\partial\Delta_t^h}{\partial i_t} =$$
$$\underbrace{\beta p^h \frac{\partial(V_{t+1}^h)}{\partial i_t} + \beta(1-p^h)\frac{\partial(V_{t+1}^\ell)}{\partial i_t}}_{\text{Constant probability case}} - \beta p^h(V_{t+1}^h - V_{t+1}^\ell)\frac{\partial\Delta_t^h}{\partial i_t} - \beta p^h\Delta_t^h\frac{\partial(V_{t+1}^h - V_{t+1}^\ell)}{\partial i_t} \tag{29}$$

Collecting the terms we can denote that in optimum, if the regime switching probabilities are endogenous, the policy maker is choosing rates to optimise dynamic welfare according to equation 10 and is also considering how this decision is influencing the switching probabilities, and how this in turn transmits to welfare.[10]

To understand the implications of equation (29) it is instructive to look at a concrete example. Similar to Adam and Woodford (2012) we assume that credibility is influenced by deviations of inflation realisations (π_t) from the target (π^*). Specifically we define the approximation of p_t^h as $p_t^h = p^h(1-\Delta_t^h)$ with

$$\Delta_t^h = \Delta^h(\pi_t - \pi^*) \tag{30}$$

Under the fairly common assumption of $\frac{\partial \pi_t}{\partial i_t} < 0$ and $V_{t+1}^h - V_{t+1}^\ell > 0$ raising rates for the case of inflation above target and reducing rates for the case of inflation below target, both increase the probability of staying in the credible regime.[11] In comparison to the case of constant switching

[10] See Appendix D for a derivation of this result based on an assumption that the endogenous probability is driven by squared deviations of inflation from the central bank's target.

[11] The first assumptions is a property of the underlying model and shared by most macroeconomic models. The second assumption is due to the property that the model $k = 0$ has a higher utility than a model with $k \neq 0$.

probabilities, the central bank responds more aggressively to inflation deviations from target. In addition to the standard stabilization criterion of the central bank, it takes into account that the rate decision influences the probabilities of switching to the alternative regime. While the optimal policy in the case of constant switching probabilities, leads to higher rates and a bias in the resulting output gap, the endogenous switching introduces a more aggressive response to surprises in inflation.

In conclusion, optimal monetary policy in a regime switching environment can be characterized as follows: First, under discretion and a constant, exogenous switching probability, the optimal policy is regime dependent and sets interest rate to offset the output gap and inflation gap. Under both regimes the optimal policy follows the standard description as in Clarida et al. (1999). A supply shock is partially accommodated, while a demand shock leads to a stronger policy response. A bias in the inflation target of the central bank gives rise to a bias in output growth, that is suboptimal from a timeless central planner's perspective.

Second, under commitment the optimal policy under constant, exogenous switching probabilities, will result in a smoothing rule, where output gap growth rates and inflation growth rates are dynamically offset, by considering the possibility of future regime switching. As in the case of discretion the rate will be higher and the output gap lower than for the case of agreement on the target.

Third, accounting for the ability of the central bank to influence the probability of switching between regimes, under benign assumptions, the optimal policy prescribes that the central bank would respond more to deviations of inflation from target to build credibility, setting rates more aggressively to lean against evidence of inflation expectations de-anchoring.

2.3 The central banker as a risk manager

If the central bank accounts for the existence of different states of the economy and the probability of switching between these regimes, it is acting as a risk manager. Similar to Kilian and Manganelli (2008) the trade-off is between the risk to price stability and the size of the losses implied by output volatility.[12] A novel element of our approach is that this risk management ap-

[12] Alternatively, the risks of too high interest rate volatility could also be evaluated in terms of recessions or periods of strong excess demand.

proach is including non-linear elements related to the regime switch which is again endogenous to the decisions of the central bank.

To bring the theoretical results from the stylized model into a more general and applied policy framework, we introduce a measure for the risks of a de-anchoring of inflation expectations, that can be evaluated on recent data or a forecast or scenario of a policy institution.

3 A quantitative approach to de-anchoring of inflation expectations

The theoretical result for the central bank to set rates more aggressively in case of emerging de-anchoring risks can be quantified in an empirical policy model. In this section, we illustrate an approach to quantify de-anchoring risks in a medium-sized DSGE model. While de-anchoring risks could also be derived from financial market information, the model based general equilibrium approach allows to evaluate the contribution of monetary policy to de-anchoring risks and provides a framework for counterfactual exercises.

We briefly revisit the state space implementation of regime switching models based on the Kim (1994) filter and propose a method for stochastic simulations of regime switching models. Then, we provide an empirical exercise based on a policy workhorse model to provide a measure of risks of de-anchoring of medium term inflation expectations.

3.1 Filtering and simulating regime switching models

We implement the regime switching using a Kim (1994) - filter, where appendix A provides the details. Here we provide the main intuition of the approach. In every period t, agents have a prior on the probability of being in regime i. They observe realized data y_t and update the Kalman filter state of each regime i. Based on the Kalman filter forecast errors of the two regimes, agents update the probability of being in one of the regimes. After the update the distribution of the two regimes is merged into a single distribution of the current state, to avoid the increase of dimensionality due to the path dependence of states.

The Kim filter (Kim, 1994) has been applied in various fields, including macroeconomics, finance, and engineering, to model systems with multiple states and estimate the parameters and transitions between those states. The filter has been found to perform well in these applications, providing more accurate estimates compared to single-regime models or models that assume fixed parameters.

Turning to the implementation of the regime switching Kalman filter, we define the following notation: where x denotes the state vector, and y the observation vector. We use the superscript i to denote the regimes.

Consider the state representation of the DSGE with measurement errors:

$$x_t^i = \mathbf{F^i} x_{t-1}^i + \mathbf{w^i}_t, \tag{31}$$

$$y_t^{MSV} = \mathbf{H^i} y_t^i + \mathbf{u}_t \tag{32}$$

Where $\mathbf{F^i}$ is identical to the reduced form VAR solution of the DSGE in regime i, $\mathbf{w^i}_t$ is the vector of regime dependent state disturbance, i.e. exogenous shocks with a regime dependent covariance matrix Q_t^i. $\mathbf{H^i}$ is the emission matrix that selects the observable states of the model, mapping the states to the data. Lastly, $\mathbf{u}$ is the measurement noise, with a covariance matrix, $\mathbf{R}$. Furthermore we specify an exogenous transition probability for the Markov-switching, that is the probability to transit from state $i \rightarrowtail j$.

$$x_{t|t}^{i(j)} = E\left[x_t | y_{1:t}, S_{t-1} = i, S_t = j\right] \tag{33}$$

Notice that the superscript in the brackets is ordered, that is the switching of regime in period t is from i to j.

To bring the application of the Kim filter into a concrete policy setting, several points need to be taken into account. First, at the time of the policy decision in t the policy makers can only base their decisions on data y_t available at that point in time. This implies that the analysis must be conducted as a real time exercise, possibly also containing a forecast available at time t. Second, the use of a forecast increase the uncertainty underlying the exercise. Both aspects will be discussed in the next section.

3.2 Conditional stochastic simulations

We propose using conditional predictive densities in a regime-switching DSGE model to assess risks surrounding forecasts. This approach relies on two main components: the stochastic nature of the simulations and the conditioning on central bank forecasts.

First, let us turn to the stochastic nature of the simulations. In this setting, stochastic simulations are a way for central banks to simulate different scenarios and see how various future paths, with possibly endogenous policy responses, affect the perceptions about the central bank's credibility. The stochastic simulations around the forecasts can effectively be interpreted as a large number of counterfactuals, each representing a different scenario. The scenarios are characterized by different assumptions about the future evolution of the economy. They capture some scenarios where prices are driven by supply- or demand shocks, some where inflation is driven by policy mistakes, and some where external factors play a key role. All counterfactuals are centered around the central bank's forecast.[13] In other words, each scenario is driven by a sequence of exogenous shocks, and the responses of agents who dynamically re-evaluate their beliefs about the credibility of the central bank. The stochastic simulations in combination with the non-linear of the regime switching evaluate wide bands around the central path, implying a comprehensive evaluation of uncertainty.

This leads to the second component, the conditioning on central bank's projections. We assume that economic agents are fully aware of the central bank's projected path for the economy and adjust their expectations accordingly. Thus the stochastic simulations are conditional on the central bank's baseline. Appendix B provides the details about the algorithm we use to generate the conditional forecasts.

Under an assumption of certainty, agents expect the projections to materialize, and consequently find no evidence toward changing their initial beliefs. However, if stochastic shocks materialize and the actual economy diverges from these forecasts, doubts can arise about the central bank's ability to stabilize inflation. In this situation of limited credibility, agents rely on the regime-switching DSGE to interpret the evolution of the economy. Based on the forecast errors of the credible regime versus the forecast errors of the de-anchored model, they evaluate

[13] A more explicit use of scenarios for policy setting is described in the report by B. Bernanke on the forecasting and policy process at the Bank of England, Bernanke (2024)

the central bank's credibility. In a linear DSGE model with a single regime, a conditional forecast would merely sum the cumulative effects of current and past structural shocks, overlaying the baseline. In contrast, a regime-switching DSGE model introduces non-linearity, as the impact of structural shocks varies with the regime, driving the perceived credibility of the central bank. Each scenario thus yields a distinct measure of central bank credibility, represented as the probability of being in a high-credibility regime. This state variable is crucial to construct the risk of de-anchoring measure, as it reflects the perceived probability of the agents to be in the de-anchored regime. It depends on the prior beliefs - the decisions of the central bank and its stock of credibility -, uncertainty - measured by the dispersion of the structural shocks and their accuracy - conditioning path - baseline projections - and ultimately the realized shocks.

3.3 A metric for de-anchoring risk

The proportion of simulated paths where the economy enters the de-anchored regime provides a measure of de-anchoring risk. We consider the economy to enter a de-anchoring episode if the perceived probability of the agents to be in the de-anchored regime exceeds 50 percent. Furthermore, we differentiate between upward and downward de-anchoring. Among the de-anchored paths, we categorize a path as upward de-anchored if the perceived inflation target is above the central bank's target. If this is not the case, the path is categorized as downward de-anchored.

In summary, conditional stochastic simulations provide a robust framework for evaluating the potential risks surrounding forecasts in regime-switching DSGE models. By generating a multitude of counterfactual scenarios that account for the stochastic nature of the economy and the conditional expectations shaped by central bank forecasts, these simulations allow us to assess how different shocks and policy responses might influence perceptions of central bank credibility.

3.4 An empirical application to the euro area

We introduce regime switching into a workhorse DSGE policy model and evaluate how the economy is switching between different regimes between 2010 to 2024. Specifically, we use the model described in Christoffel et al. (2008). The NAWM-I is a relatively standard DSGE model developed at the ECB as a tool for forecasting and policy analysis. It is build around a standard core block similar to Smets and Wouters (2003), but embedded into an international environment. It has been extensively used and fine-tuned in the projection exercises until the introduction of NAWM-II (Coenen et al. (2018)).

In the regime-switching application, the economic structure of the model under consideration is identical in both regimes, except for the perceived inflation target of the households. While this target is time-invariant and identical to the target of the central bank in the high credibility regime, the perceived target is driven by past realizations of inflation in the low credibility regime.

The main regime of the model is characterized by fully rational expectations and the central bank´s reaction function described by

$$i_t = \rho i_{t-1} + (1-\rho)\left(\pi^* + \gamma_\pi \widehat{\pi}_t + \gamma_y \widehat{y}_t\right) + \varepsilon_{i,t} \tag{34}$$

where $\widehat{\pi}_t$ is the inflation gap between actual inflation and the constant inflation target π^*, $\widehat{y}_t$ is the output gap and $\varepsilon_{i,t}$ is an i.i.d. error term.

In the alternative regime, the central bank continues to set rates according to (34), but agents do not expect the central bank to be able to achieve this target. Instead they base their decisions on a time varying inflation target. The evolution of this perceived target is driven by past perceived inflation gaps.

$$\pi_t^{*,\ell} = \pi_{t-1}^{*,\ell} + \varsigma\left(\pi_{t-1}^{(4),\ell} - \pi_{t-1}^{*,\ell}\right) + \varepsilon_{\pi^*\ell,t} \tag{35}$$

where $\pi_t^{(4),\ell}$ is the average annual inflation rate in the low credibility regime and $\varepsilon_{\pi^*,t}$ is a shock to the perceived inflation target. Under de-anchored inflation expectations the agents base their decisions on the time varying target. Higher realisations of inflation drive up the target and lead to higher price expectations in the wage and price setting decisions of firms and households. This in turn increases wages and prices and the perceived target. These negative feedback loops require a more aggressive monetary policy reaction to reign in inflation expectations and to reduce the perceived target.

Filtering and of regime switching models

Filtering the euro area data with the regime switching model provides a time series for the state variable $p_{i,j}$, the agents' perceived probability for being in each of the regimes.

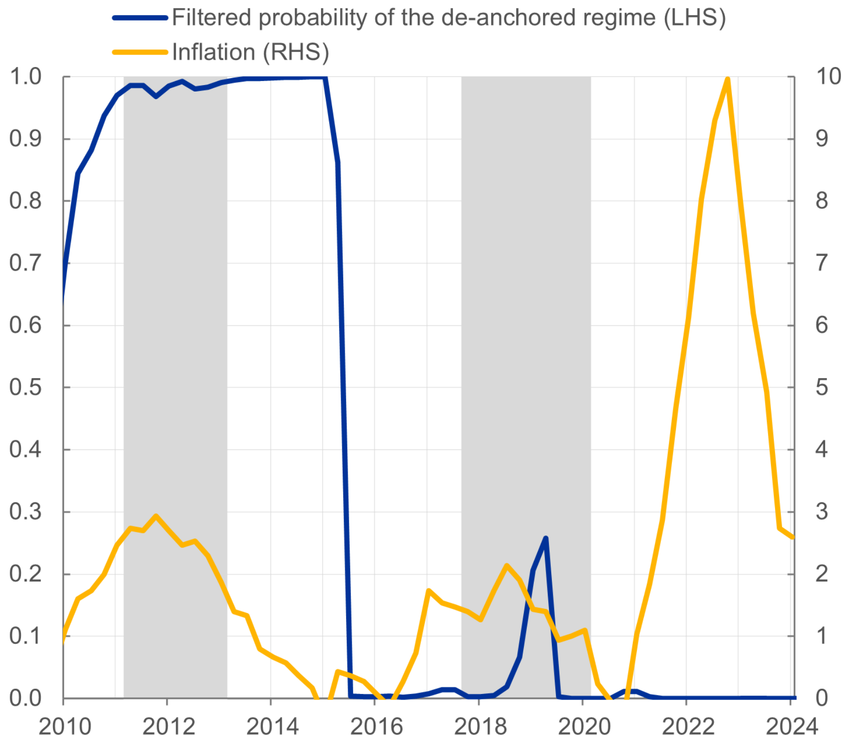
Figure 3 shows model-identified episodes of well anchored expectations and de-anchored expectations in the euro area from 2010 to 2024. There are three main drivers of de-anchoring in this framework. First, the realised data, and in particular the deviation of inflation from target. Repeatedly high, above-target (or low, below-target) inflation outcomes can increase the perceived probability of being in the de-anchored regime. If inflation is persistently deviating from target, agents loose the trust that the central bank will achieve the target in the medium run. Second, high volatility of the data supports the perception of a time-varying inflation target. Under high volatility the distribution spreads out and the probability for persistent and strong deviations of inflation from target increases.[14] Third, a slow pace in adjusting interest rates, e.g. if the central bank does not raise rates sufficiently in view of high inflation realizations, supports de-anchoring. If a central bank is reacting forcefully to deviations of inflation from target, the implied inflation stabilization is reducing the probability of de-anchoring. In addition to these drivers various modelling choice are obviously affecting the identification of de-anchoring. Especially the assumptions on the loading of the inflation gap on the perceived target, ς in (35) has a strong impact on how quickly deviations of inflation from target lead to a change in the perceived inflation target. An inflation target reacting quickly to recent inflation outcomes might favor the de-anchored model because the additional degree of freedom allows for a higher likelihood in the filtering step.

Figure 3 shows the quarterly paths from 2010-2024 of annual inflation (yellow line),and the filtered path of the perceived inflation target (blue line) evolving according to equation 35.

A first period of de-anchored inflation expectations can be identified between 2010 and 2015 (2010s episode). Driven by the deflationary episode initiated by the Lehman failure and interest

[14]For shorter episodes of high volatility the learning filter is scaling down the information content of the date, reducing the effect of volatility on de-anchoring probabilities.

Figure 3: A (recent) historical view on de-anchoring in the euro area



Notes: The blue line shows the filtered estimate of the perceived probability of the agents to be in the de-anchored regime measured on the left axis. The yellow line shows annual inflation (measured on the right scale) over the same period. The shaded episodes denotes recessionary periods according to the EABCN.

rates approaching the zero-lower bound (ZLB) and later the effective lower bound (ELB), the model identifies a disconnect between policy rates, output growth and inflation. This disconnect is favoring the model with a time-varying inflation target in providing an explanation for the episode. The 2010s episode is characterized by a slow and persistent decline in inflation rates from 2011 to 2015. The inflation target is following the decrease in headline with some time lag and stays at low levels thereafter. Due to the existence of the ELB, rates are initially decreased, but then fall short of providing further accommodative stance. Agents in the economy gradually shift to the perception that the central bank will not be able to bring back inflation to target in the medium term. Instead, the time varying inflation target provides a better fit to interpret the macroeconomic data, increasing the probability of being in the de-anchored regime. It is important to note that the underlying model is identifying the policy contribution only via the Taylor rule which is furthermore not constrained by an ELB. Accounting for the complete measures of monetary policy, especially the channel via asset market purchases and forward guidance of the central bank could reduce the prevalence of de-anchoring in this episode.[15]

It is instructive to compare this episode to the more current experience from 2021 to 2024 (2020ies episode), which is characterized by high inflation but low probabilities of de-anchoring. The degree of persistence of inflation deviations from target and the response of the central bank are two key factors, explaining low de-anchoring probabilities. It is important to note that at the beginning of the recent episode, the inflation anchor was at a very low level around 1.1% (Figure 5). Only with the strong and short-lived increase in inflation in 2021 and 2022, the perceived anchors starts to increase slowly and reaches levels above two percent only in the course on 2023, where inflation is already on a declining path. Furthermore, the recent increase in inflation has been accompanied by an unprecedented increase in interest rates, providing a clear signal that the central bank is maintaining its unchanged inflation target.

The results presented in this section are based on the filtering of the data with the regime switching DSGE model. The state variable p_t denotes the perceived probability of being in the de-anchored regime. This approach is useful to benchmark the historical results of the model, taking into account the full data set. Because of end sample problems of the smoother, these results are not very well suited to provide policy advice for an actual decision. In the next section we turn to stochastic simulations around a baseline forecast which provide a more robust and timely picture of de-anchoring.

3.5 Risk of Medium Term Expectation De-anchoring

In this section we describe a real-time exercise to evaluate the risks of de-anchoring of medium-term inflation expectations for the euro area. The method described in section 3.1 has two advantages to provide policy advice. First, the analysis is based on a real time approach, taking the actual data, nowcast and projection or scenario as the starting point for each exercise. Second, the stochastic simulations reflect the uncertainty underlying the projections and allow to derive a risk measure. In every quarter (March, June, September and December) the ECB is publishing a macroeconomic forecast.[16] At each of these data points we collect the real time

[15] It is important to note that the main driver of de-anchoring are shortfalls of closing the inflation gap, which is observable. Implicity, the non-standard policy measures are captured in the error term of the Taylor rule.

[16] The projection data is available under Macroeconomic Projection Database

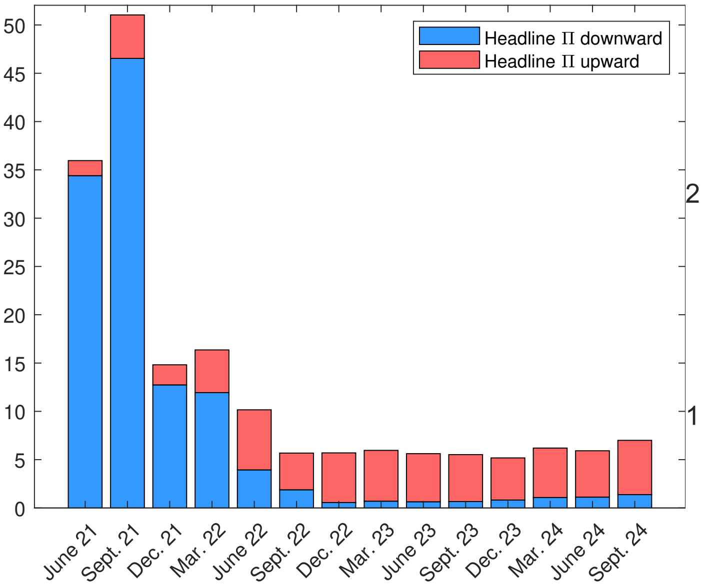
data and append it with the published forecast. This extended dataset is filtered with the regime switching model, producing filtered variables including the probability to be in the de-anchored regime. We conduct stochastic simulations in the form of conditional forecasts based on the regime-switching version of the model for 10 quarters.[17] Based on 5000 path simulations centered around the forecast path, we compute the proportion of de-anchored paths as those paths where the weight on the de-anchored regime is above 50% for at least one period, relative to the total number of simulations. We are furthermore differentiating between upward de-anchoring (red bars) and downward de-anchoring (blue bars). A de-anchored path is classified as upward de-anchored if the perceived inflation target of that simulation remains above the 2% target of the ECB. If the perceived target falls below the 2% target the path is classified as downward de-anchored.[18]

Figure 4 shows the model implied risks of de-anchoring for various projection exercises of the eurosystem. The highest risks of de-anchoring are identified in the course of 2021. In 2021 inflation had been undershooting the target for around 8 years, driving down the perceived target of the central bank. In this situation a time varying inflation target provided a better fit to the observed data, leading to increasing weights on the de-anchored regime. During this period, the perceived target of households was clearly and persistently below 2%, implying that downward de-anchoring was the dominant risk of de-anchoring. In the second half of 2021 inflation started to increase, which reduced the downward de-anchoring risks strongly. Since the perceived target is a slow moving variable the upward de-anchoring risks increased to a lesser extend.

[17]The choice for 10 quarters is given by the shortest horizon. The horizon is given by the nowcast plus the forecast over the next two years. Consequently, the shortest sample over the year occurs in September. In September the nowcast is the 3^{rd} quarter of the current year and the forecast spans from the 4^{th} quarter over the two subsequent years.

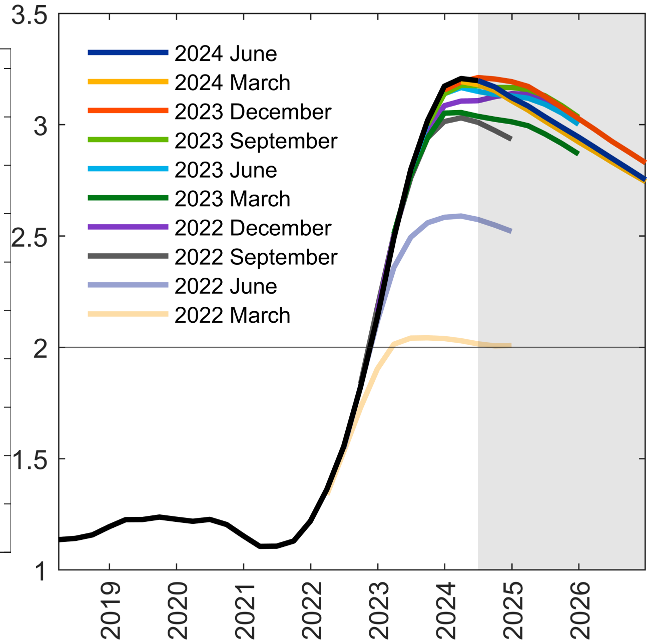
[18]For those paths where the perceived target crosses the 2% line, the sign of maximum deviation from 2% is taken as the criterion to classify downward or upward de-anchoring.

Figure 4: Risks of de-anchoring of medium term inflation expectations
(percentages)



Sources: Authors' calculations.
Notes: The charts show the risk of de-anchoring for the projections from June 2021 to June 2023. The blue bars indicate downward, the red bars indicate upward de-anchoring.
Latest observations: 2024Q2.

Figure 5: Perceived inflation target in the projections from June 2021 to June 2024
(year-on-year percentage points)



Sources: Authors' calculations.
Notes: The charts show the perceived inflation target from June 2021 to June 2023.
Latest observations: 2024Q2.

3.6 Drivers of de-anchoring risks

Figure (4) gives an indication how the evolution of the economy is driving de-anchoring risk between 2021 and 2024. In addition to the underlying data, there are further factors and modeling choices that drive the risks of de-anchoring.

3.6.1 Impact of baseline projections

The conditional stochastic simulations are conducted around a baseline or a baseline projection. Since the risks of de-anchoring are state dependent, the baseline itself is influencing the evolution of the filtered probabilities of the de-anchored regime strongly. These projections provide a conditional path for key variables like inflation and output. If the baseline projection anticipates persistent deviations of inflation from the target, even under the central bank's policy actions, it will contribute to an increase in the perceived probability of de-anchoring. They can also lead to perceptions of de-anchoring if the relationship between the projected variables are at odds with historical regularities observed in the data. For instance, even if the baseline projections anticipate a closure of the inflation gap, while at the same time energy price pressures are expected to prevail, and the exchange rate is expected to depreciate then the disconnect of inflation from its components will be attributed to de-anchoring.

In September 21, starting from the background of the low inflation period (with mean inflation of 1.0% between 2015q1 and 20221Q2) with interest rates close to the effective lower bound, forecasters at that time assumed only a gradual increase in inflation, staying below 2 percent over the full projection horizon. This outlook was similar in the financial market, who expected the rate to stay only moderately above the ELB level with a peak of -0.5%. In this specific situation the model identified high risk of de-anchoring, with downside risks clearly dominating, see row 'Sep 21' in table (2). To illustrate the impact of the baseline, we re-simulate the 'Sept. 21' exercise, but assume that the forecast of all variables coincided with the ex-post realization of those data. The actual realization of data implied a significantly higher path for inflation and also for policy rates. Row 'ex-post' data in table (2), shows that downward de-anchoring risks decline strongly in comparison to the version using real time data an forecast. However, the relative frequency of upward de-anchored paths in the total number of de-anchored path increases markedly. The high inflation in the ex-post data version imply that a lower proportion of paths enters the territory below 2%. This reduced downward de-anchoring risks. However, the perceived target is persistent and increases only gradually, preventing mounting upside risks to inflation. If the perceived target is below 2% while actual inflation is above 2% the de-anchored regime is not providing a better explanation of observed data, than the constant target regime.

3.6.2 Central bank reaction

The discussion in section (2) shows that the optimal policy under de-anchoring risks should be more responsive to deviations of inflation from target than in the one-regime case. Similarly, in the stochastic simulations a more responsive central bank will stabilize the economy more quickly, contributing to lower de-anchoring risks. Note that in the stochastic simulations of

Table 2: Sensitivities of de-anchoring risks to economic fundamentals

	pre-sample	evaluation sample				
	mean inflation	mean inflation	peak pol. rate	risk deanchoring total	up	down
Sept. 21	1.0%	1.7%	-0.5%	56	8	48
Ex-post data	1.0%	5.8%	4.0%	16	6	10
Rate setting	1.0%	5.8%	-0.5%	30	20	10
Stock of Cred.	2.9%	5.8%	4.0%	61	59	2
Uncertainty	1.0%	5.8%	4.0%	53	16	37

Notes: All moments are based on 5000 path simulations of the model described above. The entry 'Sept. 21' stands for the real time exercise of September 21. 'Ex-post' data assumes that forecasters in September 21 knew the ex-post realizations of the data over the full horizon. The entry 'Rate setting' evaluates the counterfactual of 'ex-post' but assuming that rates stay at the path assumed in the September 21 forecast. The 'stock of credibility' gives a scenario based on higher inflation in the pre-sample and consequently lower credibility. The 'Uncertainty' entry shows the results for the 'ex-post' version but shocks being scaled up by a factor of 5.

section (3.4) the financial markets expectations of the interest rate path are taken as the baseline path for interest rates. A more aggressive policy will stabilize the economy via the expectational channel. Keeping rates at lower levels, will increase de-anchoring risks.

To illustrate the role of interest rate setting we construct a scenario 'Rate setting' as a counterfactual to the entry 'ex-post data' in graph 4. We assume again, that forecasters had perfect knowledge of the ex-post realization of data, but policy rates would stay as low as expected in 'Sept. 21'. In monetary policy had looked fully through the inflation surge, inflation would have increased significantly and reduced the weight on the well-anchored regime, giving rise to sizable upside de-anchoring risks. The overall de-anchoring risks increase to around 30 percent. The high downward de-anchoring risks from 'Sept 21' are strongly reduced and upward risks increase markedly, contributing two-thirds of total de-anchoring risks.

3.6.3 Stock of credibility

The initial level of trust that agents place in the central bank's ability to control inflation forms the foundation for the probability assigned to the regimes. A central bank with a strong track record of maintaining price stability starts with a higher stock of credibility and thus a lower probability for the de-anchored regime. Conversely, if the central bank has previously struggled with inflation, agents will be more inclined towards the de-anchored regime ex-ante. This initial condition reflects the accumulated credibility (or lack there of) of the central bank.

Preceding the inflation surge, the euro area went through a long period of low inflation and rates close to the ELB. When entering the inflation surge the credibility was reduced due to several years of inflation below target. In addition to the reduced stock of credibility, the perceived target was significantly below two percent. The low target did not help to explain the high inflation realization, implying that the model simulations are predominantly identifying de-anchoring in the lower tail of inflation realisations, implying the high downward de-anchoring risks shown in the right columns of table (2) for 'Sept. 21'.

We conduct a counterfactual simulation ('Stock of credibility' in table (2)), where we modify the data prior to the simulation horizon. Specificially, we feed the model with positive demand shocks to engineer a demand driven inflation increase, pushing average inflation in the 'pre-sample' (2015q1 to 2021q2) to 2.9% annual inflation. The higher inflation drives up the perceived inflation target and reduces credibility of the central bank at the same time. As a result of these two factors, de-anchoring risks to the upside increase considerable (row 'Stock of credibility') over the simulation horizon, which includes the inflation surge. De-anchoring risks increase if the stock of credibility is low and the perceived target deviates from 2% into the same direction as the inflation surprises. The calibration of the evolution of the perceived target is also affecting the identification of de-anchoring risks. The results of the paper are produced with a persistence parameter (ς in equation (35)) of 0.98. In appendix (E) we report the sensitivity of de-anchoring risk, using the calibration of Coenen and Schmidt (2016).

3.6.4 Impact of uncertainty

Heightened economic uncertainty, reflected in a higher volatility of structural shocks hitting the economy, may contribute to a higher evidence for de-anchoring. This is partially explained by the property that the de-anchored regimes has an additional state variable allowing for higher flexibility to explain volatile data. When agents face a turbulent environment with wide-range of potential outcomes, they are more likely to doubt the central bank's ability to steer inflation towards its target. This uncertainty can stem from factors like volatile energy prices, unexpected supply chain disruptions, or unexpected geopolitical events. On the other hand if the uncertainty is anticipated, then it can reduce the information content of the de-anchoring signals. When large shocks fall short of the scale they were expected to be drawn from, then heightened uncertainty can be reaffirming the central bank's credibility by discounting evidence of de-anchoring more. The row 'Uncertainty' illustrates this property by re-simulating the 'Sept. 21' exercise, scaling the shocks by a factor of 5.

3.6.5 Impact of realized shocks

Furthermore, the actual shocks that hit the economy ultimately determine the trajectory of perceived beliefs. If these shocks align with the baseline projections, the perceived probability of de-anchoring will likely remain unchanged. However, if unexpected shocks push inflation persistently away from the target, despite the central bank's efforts, agents will update their beliefs, leading to a higher de-anchored regime probability. For instance, a sudden surge in energy prices not anticipated by the baseline projection could trigger such an update.

The de-anchoring probabilities are closely linked to the forecast errors. If the credible regime's sequential forecast, given the realization of the shocks, implies smaller forecast errors than the ones of the de-anchored regime, the evidence for de-anchoring is reduced and the central bank increases its stock of credibility. If the realizations of the shocks change the relative forecast performance in favor of the de-anchored regime, the evidence for de-anchoring increases. It is important to note that changes to the perception of the regimes take place only gradually. The required evidence for a regime switch needs to be sufficiently large and persistent. Generally speaking, in situations of high uncertainty, large shocks might generate economic states inconsistent with a credible central bank, suggesting a potential loss of credibility.

3.7 A counterfactual scenario: looking through external price shocks

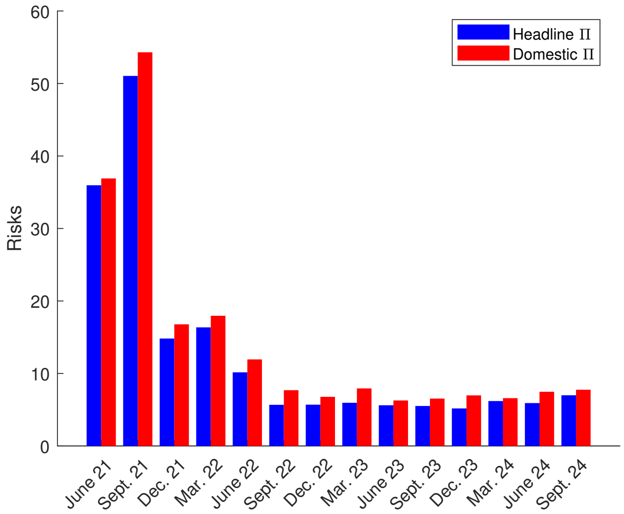
The optimal policy exercise in section 2 showed that under a mounting risk of a de-anchoring of inflation expectations the central bank should react more forcefully against deviations of inflation from target. We illustrate this property by revisiting the exercise illustrated in figure 4. We change the central bank's inflation measure from headline consumer prices to the value of domestically produced goods and services (GDP deflator). The central bank´s reaction function can then be described by

$$i_t \quad = \quad \rho i_{t-1} + (1-\rho)\left(\pi^* + \widehat{\pi}_t^{core} + \widehat{y}_t\right) + \varepsilon_{i,t} \tag{36}$$

where $\widehat{\pi}_t^{core}$ is the inflation gap between GDP deflator inflation and the constant inflation target π^*, $\widehat{y}_t$ is the output gap and $\varepsilon_{i,t}$ is an i.i.d. error term.

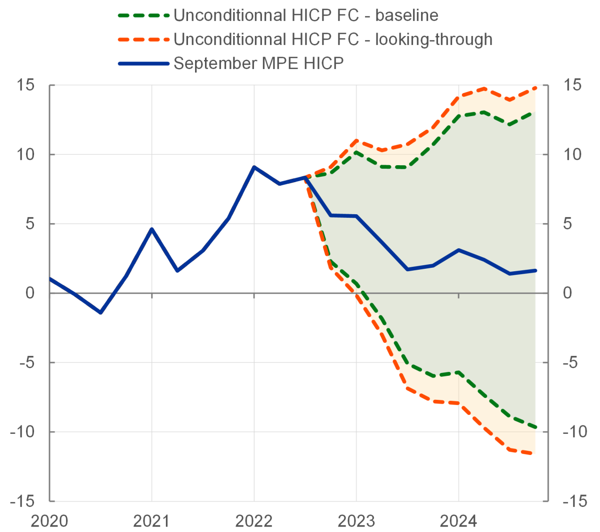
Figure (6) shows that the overall risks of de-anchoring increase for all exercises, but to a limited extend. If the central bank is looking through price pressures originating from imported goods, it is tolerating more pronounced deviations of headline inflation from target. The deviations can lead to a higher weight on the de-anchored regime, increasing also the risks of de-anchoring. The size of the increase in risks is however limited. This is due to the medium-term orientation of the ECB, implying a certain degree of looking-through temporary shocks. This medium-term orientation is implicitly captured in the estimated coefficients of the Taylor rule. For more persistent, foreign price shocks such as the shocks occurring after the Russian invasion of Ukraine, the transmission to domestic inflation was relatively fast and substantial. This high degree of transmission implies that also for the case of targeting domestic price inflation, rates are increased substantially.

Figure 6: Risks of (upward and downward) inflation de-anchoring: Headline Inflation versus Inflation of domestically produced goods and services
(percentages)



Sources: Authors' computations.
Notes: The red bars denote upward de-anchoring, and the blue bars denote downward de-anchoring. The simulations are based on a regime-switching version of NAWM-I, where the credible regime is defined as the estimated version of the NAWM-I with a fixed inflation target, the de-anchored regime is characterized with a time-varying inflation target. The share of de-anchoring paths is based on 5000 simulations around the forecast baseline. Looking-through denotes a scenario where the Taylor rule responds to core inflation (GDP deflator).
Latest observations: 2024Q3.

Figure 7: Uncertainty around the September 2022 forecast inflation projections
(year-on-year percentage points)



Sources: Authors' computations.
Notes: The chart shows the 95% credible interval around the September 2022 forecast baseline. The uncertainty bands are based on conditional forecast of the regime-switching NAWM-I. The baseline denotes a model with the central bank responding to deviations from a fixed target and with agents' beliefs about the inflation target switching between credible and de-anchored regimes. Looking-through denotes a model with the central bank responding to deviations of core inflation from the fixed target, also allowing for de-anchoring of inflation expectations.
Latest observations: 2022Q2
.

Figure 7 shows that the uncertainty, measured as the 95% interval of the stochastic simulations, increases for the looking-through case. This increase in uncertainty is driven by the more frequent cases of the de-anchored regime, which leads to more extreme paths of inflation.

4 Conclusions

If a central bank cannot counteract the price pressures from a foreign shock, looking-through this shock can be welfare maximizing. In the conduct of actual monetary policy this is mirrored in concepts of the medium-term orientation of monetary policy or notions of average inflation targeting. Both from a theoretical as well as practical point of view, looking-through is advisable as long as inflation expectations are firmly anchored. In this paper we analyze the consequences of a possible de-anchoring of medium term inflation expectations, for the conduct of monetary policy from a theoretical as well as from a quantitatice perspective.

From a normative perspective we find that the possibility that the economy might switch into a de-anchored regime, changes the optimal monetary policy prescription in comparison to the case of a model without regime switches. In the regime switching case the optimality criteria under commitment are extended to include the impact of the current interest rate decision on the probability of switching to another regime. The implied rule under regime switches implies that the central bank in the credible regime chooses a more aggressive policy. This reflects the possibility of switching to the low credibility regime.

A further increase in the response to deviations of inflation from target is found for the case of endogenous switching. In this case the interest rate is used to stabilize and to lean against a possible switch to the low credibility case.

From a positive perspective we use a canonical policy model for the euro area and implement a regime switching version, allowing to filter for the probability of being in the respective regime. Analysing the period between 2010 and 2024 in the euro area a de-anchored regime is detected for the period between 2011 and 2015. The inflation surge in 2021/2022 is accompanied by a strong increase in interest rates and is not detected as a de-anchored regime.

Furthermore, we introduce a quantitative approach to evaluate risks of de-anchoring around a baseline or a forecast. By means of stochastic simulations of the regime switching model centered around the baseline a measure of risks of de-anchoring can be calculated. The real time exercise allows to assess the state-dependent risks of inflation de-anchoring. For the ELB episode in the euro area between 2015 and 2020 we find pronounced risks of de-anchoring, while for the post-covid, with a supply side driven inflation surge, the risks maintain contained, because of monetary policy moving towards an increasingly tighter stance. The evolution of de-anchoring risks over time can give an indication for more forceful policy in times of increasing risks of de-anchoring.

In a counterfactual policy exercise, where the central bank follows a 'looking-through oil price driven inflation' policy we show that the risks of de-anchoring increase. This increase is due to the central bank tolerating longer periods of headline inflation above target, if these deviations are driven by external factors. This result is in line with the theoretical results that a less forceful reaction to headline inflation leads to higher de-anchoring risks.

The proposed method can serve as a complement to measures of de-anchoring based on financial market data or surveys. In comparison to those measures, the model based exercise allows for a structural interpretation and to conduct policy counterfactuals.

Our methods also relate to the recent tendency in monetary policy to account more explicitly for uncertainty in the form of scenarios, as proposed in the Bernanke-report for the Bank of England. When uncertainties around the baseline projection are illustrated by counterfactual scenarios, the calculation of de-anchoring risks around the baseline and the scenarios, can be a useful extension to inform monetary policy decisions.

Future research could provide a more comprehensive evaluation of various policy alternatives with respect to the implied risks of a de-anchoring of inflation expectations.

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A Appendix: Switching Kálmán filter

In what follows, we will discuss the main steps needed to derive updating of the a-priori state probabilities. Inference is filtering only - the probability distribution of a switch happening at time t depends only on past data, i.e. $1:t$. For a full discussion of the switching Kálmán filter please consult Kim (1994); Murphy (1998); Hamilton (1989b) and additional details on Markov-switching DSGE models see Davig and Leeper (2007, 2010); Farmer et al. (2009); Foerster et al. (2016); Maih (2015) .

Let us define a notation: x is the states, while y the observations. Denote the regimes with i.

Consider the state representation of the DSGE with observable noise on the MSV states:

$$x_t^i = \mathbf{F^i} x_{t-1}^i + \mathbf{w^i}_t, \tag{37}$$

$$y_t = \mathbf{H^i} x_t^i + \mathbf{u}_t \tag{38}$$

Where x_t^i is the state vector given belief i, either forward of backward-looking, $\mathbf{w^i}_t$ is the exogenous state disturbance. Denote its covariance matrix with Q_t^i. $\mathbf{H^i}$ is the emission matrix that selects the observables of the model. Furthermore $\mathbf{u}$ is the measurement noise with a covariance matrix that is usually denoted by $\mathbf{R}$. Not to confuse the mean squared error matrix under the beliefs, and I will express the measurement error covariance matrix with $\mathbf{U}$.

Furthermore one needs to specify an exogenous transition probability from state matrix Z $i \rightarrowtail j$. For example, we assume a highly persistent exogenous state transition probability of the form:

$$Z = \begin{pmatrix} 0.9999 & 0.0001 \\ 0.0001 & 0.9999 \end{pmatrix} \tag{39}$$

Introducing the notation:

$$y_{t|t}^{i(j)} = E\left[x_t | y_{1:t}, S_{t-1} = i, S_t = j\right] \tag{40}$$

Notice that the superscript in the brackets is the switching of regime from i to j in period t. The Equation 40 tells, what the value of the full state is given the (full) history of the observables if it switches from regime i to j.

The switching Kálmán filter pass will be the following: First, the state distribution is inherited. It is all possible combination of states $x_{t|t-1}^{i(j)}$ and their respective covariance matrix based on information from $t-1$. The indices of states are looped over before progressing to the next step of the filter. The notation below exemplifies the filter as conditional on being in i switching to the next regime j. If the index is the same, then no regime switch takes place, if it is different it represents switching. As in the Kálmán filter, the first step is called the prediction:

$$x_{t|t-1}^{i(j)} = \mathbf{F^j} x_{t-1}^{i}, \tag{41}$$

$$Q_{t|t-1}^{i(j)} = \mathbf{F^j} Q_{t-1}^{i} (\mathbf{F^j})' + Q_{t-1}^{j}. \tag{42}$$

Then, we compute the Kálmán gain given switching:

$$K^{i(j)} = Q_{t|t-1}^{i(j)} (\mathbf{H^j})' (\mathbf{H^j} Q_{t|t-1}^{i(j)} (\mathbf{H^j})' + \mathbf{U}) \tag{43}$$

Using the gain update one can generate the nowcast, i.e. posterior of the state and state covariance matrix given information t:

$$x_{t|t}^{i(j)} = x_{t|t-1}^{i(j)} + K^{i(j)} (y_t - \mathbf{H^j} x_{t|t-1}^{i(j)}); \tag{44}$$

$$Q_{t|t}^{i(j)} = (I - K^{i(j)} \mathbf{H^j}) Q_{t|t-1}^{i(j)}; \tag{45}$$

With the nowcast, the likelihood of data given $S_t = j$ and $S_{t-1} = i$ can be computed that is the object of my application of the filter:

$$e_t^{i(j)} = y_t - \mathbf{H^j} x_{t|t-1}^{i(j)}, \tag{46}$$

$$L_t^{i(j)} = \sqrt{det(\mathbf{H^j} Q_{t|t-1}^{i(j)} \mathbf{H^j}' + \mathbf{U})} \cdot exp^{-\frac{1}{2}\Sigma\left(e_t^{i(j)}\left(\mathbf{H^j} Q_{t|t-1}^{i(j)} \mathbf{H^j}' + \mathbf{U}\right) e_t^{i(j)}\right)} \tag{47}$$

Finally one can update the a priori probabilities $Pr(S_t = i|t-1)$ using the following algorithm for all $i, j \in \{1,2\}$ and all t:

$$Pr(S_t = j|t, S_{t-1} = i) = \frac{L_t^{i(j)} Z(i,j) Pr(S_t = i|t-1)}{\sum_{i\in\{1,2\}} \sum_{j\in\{1,2\}} L_t^{i(j)} Z(i,j) Pr(S_t = i|t-1)} \tag{48}$$

$$Pr(S_t = j|t) = \sum_{i\in\{1,2\}} Pr(S_t = j|t, S_{t-1} = i) \tag{49}$$

The final collapsing step assures that states are merged from across the regimes with the weighted probabilities:

$$x_{t|t}^{j} = \sum_{i\in\{1,2\}} x_{t|t}^{i(j)} \cdot \frac{Pr(S_t = j|t, S_{t-1} = i)}{Pr(S_t = j|t)}, \tag{50}$$

$$Q_{t|t}^{j} = \frac{Pr(S_t = j|t, S_{t-1} = i)}{Pr(S_t = j|t)} \left(Q_{t-1}^{j} + \left(x_{t|t}^{i(j)} - x_{t|t}^{j} \right) \left(x_{t|t}^{i(j)} - x_{t|t}^{j} \right)' \right). \tag{51}$$

There are two key points to consider. First, the exogenous state state transition matrix Z. It scales the likelihood and regulates switching. This is important as switching Kálmán filters have been documented to show instability of regimes and display way too many jumps. However over-regularizing the switching, and imposing an identity matrix, eliminates changing of regimes entirely. Therefore having a reasonable yet persistent exogenous regime dynamics is preferred. This is implemented with the calibration of entries in Z. Second,the role of the observation error covariance matrix, $\mathbf{U}$. It is added to the (observation space compressed) state variance matrix, when computing the likelihood. That is, the variation of the data is either driven by the model or the measurement error. It's relative size and possible correlation structure with that of the state covariance matrix is key in determining switching.

B Appendix: Counterfactual Simulations in MS-DSGEs with Extended Path Methods

In what follows we review how conditional predictive densities of the regime-switching DSGE can be constructed to construct model consistent counterfactuals.

Conditional forecasting involves the forecast of endogenous variables conditional on a certain path for a subset of some endogenous or exogenous variables. In practice the endogenous variables are often observables, but for policy purposes they may involve scenarios for unobserved state variables as well. When treating the external information one needs to take a stance if one assumes the conditioning information to be precise, leading to *hard conditions* or represent a range for the conditioning paths, leading to *soft conditions* (Waggoner and Zha, 1999; Warne, 2022). In what follows we will discuss hard conditioning and leave soft conditioning to future research.

We follow Negro and Schorfheide (2013)'s *News* assumption to capture the condition information. In general the forecasts (y_{T+i}) and conditioning information (z_{T+i}) for forecast horizon $i = 1, 2, ..., g$ can be related with an error term:

$$y_{T+i} = z_{T+i} + \eta_{T+i} \tag{52}$$

According to the *News* assumption the conditioning information is independent of the error term, that is $z_{T+i} \perp \eta_{T+i}$, which implies that z_{T+i} is the conditional expectation of y_{T+i} given the data and the news, external information[19].

Assuming that no additional information beyond the news is useful predicting y_{T+i}, we can denote all conditioning impact with $\tilde{y}_{T+i}$. Allowing to represent the predictive density as follows:

$$p(y_{T+i}|Y_{1:T}, Z) = \int_{\theta} \left[\int_{\tilde{y}_{T+i}} p(y_{T+i}|\tilde{y}_{T+i}, Y_{1:T}, \theta) p(\tilde{y}_{T+i}|, Y_{1:T}, Z) d\tilde{y}_{T+i} \right] p(\theta|Y_{1:T}) d\theta, \tag{53}$$

where all conditioning information is Z. In other terms we can partial out the role of the external

[19]One can make the assumption that the news is fully known in period T, resulting in hard conditioning, or that there is uncertainty about its value, $\eta_{T+i} \sim N(0, \sigma^2_{eta_i})$

information in form of news on the conditional density y_{T+i} through its impact on $\tilde{y}_{T+i}$: Having controlled for the information in the conditioning we can fully characterize the predictive distribution[20].

We can then use the following algorithm to generate draws from the predictive distribution conditional on external information:

Algorithm 1: Draws from the Predictive Distribution of a regime-switching DSGE Conditional on External Information

for $j \in \{1, 2, ..., n_{sim}\}$ *draws from the posterior* **do**
- Draw $\theta^{(j)}$ from the posterior distribution $p(\theta|Y_{1:T})$;
- Use the regime-switching Kalman Filter to compute the posterior probability of the regimes and the state distribution mean and variance in period T;
- **for** $i \in \{1, 2, ..., g\}$ *periods of conditioning horizons* **do**
  - Generate the draw $\tilde{y}^{(j)}_{T+i}$ for the conditioned variable from the $p(\tilde{y}_{T+i}|Y_{1:T}, Z)$, assuming $\eta_{T+i} \sim N(0, \sigma^2_{\eta_i})$;
  - Treating $\tilde{y}^{(j)}_{T+i}$ as data use the regime-switching Kalman Filter to update the regime probabilities π_{T+i} and the conditional state distribution $p(s_{T+i}|\theta^{(j)}, Y_{1:T}, Z)$.
  - Draw a s_{T+i} from $p(s_{T+i}|\theta^{(j)}, Y_{1:T}, Z)$.
- **end**
- **for** $ii \in \{g+1, g+2, ..., H\}$ *unconditioned forecast horizons* **do**
  - Draw innovations for ε_{T+ii} for the structural shocks and iterate the state transition for each regime using the regime dependent state transition equation.
  - Aggregate the regimes using prior regime probabilities π_{T+ii-1}.
  - Run the regime-switching Kalman filter and update the regime probabilities to π_{T+ii}.
- **end**

end

[20]This rests on the assumption that the parameter posterior $p(\theta)$ remains unchanged, that is we disregard the information content of the external information with respect to the model parameters.

C Appendix: Derivation of the Optimal Monetary Policy Rule under Commitment

The social planner minimizes the expected loss

$$\mathscr{L} = E_t \sum_{s=0}^{\infty} \beta^s \left\{ -\frac{1}{2} \left[\left(\widehat{\pi}_{t+s}^{\{h,\ell\}} - \pi^{*,\{h,\ell\}} \right)^2 + \lambda \left(\widehat{y}_{t+s}^{\{h,\ell\}} \right)^2 \right] \right\}, \tag{54}$$

where, $\pi^{*,h} = \pi^{*,CB}, \pi^{*,l} = \pi^{*,CB} -$ subject to the New Keynesian constraints in two regimes. The constraints are given by:

$$\widehat{\pi}_t^h = \kappa \widehat{y}_t^h + \beta [p^h \widehat{\pi}_{t+1}^h + (1-p^h)\widehat{\pi}_{t+1}^\ell] + u_t \tag{55}$$

$$\widehat{y}_t^h = [p^h \widehat{y}_{t+1}^h + (1-p^h)\widehat{y}_{t+1}^\ell] + \frac{1}{\sigma}[p^h \widehat{\pi}_{t+1}^h + (1-p^h)\widehat{\pi}_{t+1}^\ell - i_t^h + r_t^n] + g_t \tag{56}$$

$$\widehat{\pi}_t^\ell = \kappa \widehat{y}_{t+1}^\ell + \beta [p^\ell \widehat{\pi}_{t+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+1}^h] + u_t \tag{57}$$

$$\widehat{y}_t^\ell = [p^\ell \widehat{y}_{t+1}^\ell + (1-p^\ell)\widehat{y}_{t+1}^h] + \frac{1}{\sigma}[p^\ell \widehat{\pi}_{t+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+1}^h - i_t^\ell + r_t^n] + g_t \tag{58}$$

The policy problem under timeless commitment has the following objective:

$$\begin{aligned}
\mathscr{L} = E_t \sum_{s=0}^{\infty} & -\frac{1}{2} \left(\left(\widehat{\pi}_{t+s}^{\{h,\ell\}} - \pi^{*,\{h,\ell\}} \right)^2 + \lambda \left(\widehat{y}_{t+s}^{\{h,\ell\}} \right)^2 \right) \\
& + \mu_{t+s}^{\pi^h} [\widehat{\pi}_{t+s}^h - \kappa \widehat{y}_{t+s}^h - \beta [p^h \widehat{\pi}_{t+s+1}^h + (1-p^h)\widehat{\pi}_{t+s+1}^l] - u_{t+s}] \\
& + \mu_{t+s}^{y^h} [\widehat{y}_{t+s}^h - [p^h \widehat{y}_{t+s+1}^h + (1-p^h)\widehat{y}_{t+s+1}^l] \\
& - \frac{1}{\sigma}[p^h \widehat{\pi}_{t+s+1}^h + (1-p^h)\widehat{\pi}_{t+s+1}^l - i_{t+s}^h + r_{t+s}^n] + g_{t+s}] \\
& + \mu_{t+s}^{\pi^\ell} [\widehat{\pi}_{t+s}^\ell - \kappa \widehat{y}_{t+s}^l - \beta [p^\ell \widehat{\pi}_{t+s+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+s+1}^h] - u_{t+s}] \\
& + \mu_{t+s}^{y^\ell} [\widehat{y}_{t+s}^\ell - [p^\ell \widehat{y}_{t+s+1}^\ell + (1-p^\ell)\widehat{y}_{t+s+1}^h] \\
& - \frac{1}{\sigma}[p^\ell \widehat{\pi}_{t+s+1}^\ell + (1-p^\ell)\widehat{\pi}_{t+s+1}^h - i_{t+s}^\ell + r_{t+s}^n] + g_{t+s}]
\end{aligned} \tag{59}$$

The first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial \widehat{\pi}^h_{t+s}} : E_t[\beta^s[(\widehat{\pi}^h_{t+s} - \pi^{*,CB}) + \mu^{\pi^h}_{t+s}] - \beta^{s-1}[\beta p^h \mu^{\pi^h}_{t+s-1} + \beta(1-p^\ell)\mu^{\pi^\ell}_{t+s-1}]$$
$$- \beta^{s-1}\mu^{y^h}_{t+s-1}\frac{1}{\sigma}p^h - \beta^{s-1}\mu^{y^\ell}_{t+s-1}\frac{1}{\sigma}(1-p^\ell)] = 0 \quad (60)$$
$$\frac{\partial \mathscr{L}}{\partial \widehat{y}^h_{t+s}} : E_t[\beta^s[\lambda \widehat{y}^h_{t+s} + \mu^{y^h}_{t+s}] - \beta^{s+1}\mu^{\pi^h}_{t+s+1}\kappa - \beta^{s-1}\mu^{y^h}_{t+s-1}p^h - \beta^{s-1}\mu^{y^\ell}_{t+s-1}(1-p^\ell)] = 0 \quad (61)$$
$$\frac{\partial \mathscr{L}}{\partial \widehat{\pi}^\ell_{t+s}} : E_t[\beta^s[(\widehat{\pi}^\ell_{t+s} - -\pi^{*,CB}) + \mu^{\pi^\ell}_{t+s}] - \beta^{s-1}[\beta p^\ell \mu^{\pi^\ell}_{t+s-1} + \beta(1-p^h)\mu^{\pi^h}_{t+s-1}]$$
$$- \beta^{s-1}\mu^{y^\ell}_{t+s-1}\frac{1}{\sigma}p^\ell - \beta^{s-1}\mu^{y^h}_{t+s-1}\frac{1}{\sigma}(1-p^h)] = 0 \quad (62)$$
$$\frac{\partial \mathscr{L}}{\partial \widehat{y}^\ell_{t+s}} : E_t[\beta^s[\lambda \widehat{y}^\ell_{t+s} + \mu^{y^\ell}_{t+s}] - \beta^{s+1}\mu^{\pi^\ell}_{t+s+1}\kappa - \beta^{s-1}\mu^{y^\ell}_{t+s-1}p^\ell - \beta^{s-1}\mu^{y^h}_{t+s-1}(1-p^h)] = 0 \quad (63)$$

The index s can be dropped assuming the commitment is from a timeless perspective. Then the FOCs can be written as:

$$(\widehat{\pi}^h_t - \pi^{*,CB}) + \mu^{\pi^h}_t - p^h\mu^{\pi^h}_{t-1} - (1-p^\ell)\mu^{\pi^\ell}_{t-1} - \frac{p^h}{\sigma\beta}\mu^{y^h}_{t-1} - \frac{(1-p^\ell)}{\sigma\beta}\mu^{y^\ell}_{t-1} = 0 \quad (64)$$
$$\lambda \widehat{y}^h_t + \mu^{y^h}_t - \beta\kappa\mu^{\pi^h}_{t+1} - \frac{p^h}{\beta}\mu^{y^h}_{t-1} + \frac{1-p^\ell}{\beta}\mu^{y^\ell}_{t-1} = 0 \quad (65)$$
$$(\widehat{\pi}^\ell_t - -\pi^{*,CB}) + \mu^{\pi^\ell}_t - p^\ell\mu^{\pi^\ell}_{t-1} - (1-p^h)\mu^{\pi^h}_{t-1} - \frac{p^\ell}{\sigma\beta}\mu^{y^\ell}_{t-1} - \frac{(1-p^h)}{\sigma\beta}\mu^{y^h}_{t-1} = 0 \quad (66)$$
$$[\lambda \widehat{y}^\ell_t + \mu^{y^\ell}_t] - \beta\mu^{\pi^\ell}_{t+1}\kappa - \frac{p^\ell}{\beta}\mu^{y^\ell}_{t-1} + \frac{1-p^h}{\beta}\mu^{y^h}_{t-1} = 0 \quad (67)$$

Solving for the contemporaneous shadow costs of the Phillips Curves gives us the following:

$$\mu^{\pi^h}_t = -\left(\widehat{\pi}^h_t - \pi^{*,CB}\right) + p^h\,\mu^{\pi^h}_{t-1} + (1-p^\ell)\,\mu^{\pi^\ell}_{t-1} + \frac{p^h}{\sigma\beta}\,\mu^{y^h}_{t-1} + \frac{(1-p^\ell)}{\sigma\beta}\,\mu^{y^\ell}_{t-1} \quad (68)$$
$$\mu^{\pi^\ell}_t = -\left(\widehat{\pi}^\ell_t - k - \pi^{*,CB}\right) + p^\ell\,\mu^{\pi^\ell}_{t-1} + (1-p^h)\,\mu^{\pi^h}_{t-1} + \frac{p^\ell}{\sigma\beta}\,\mu^{y^\ell}_{t-1} + \frac{(1-p^h)}{\sigma\beta}\,\mu^{y^h}_{t-1} \quad (69)$$

The shadow cost of the IS curve are zero, except for the initial period under commitment from a timeless perspective (Woodford, 2004) thus it is sufficient to focus on the shadow cost related the Phillips Curves. Realize that we have recurring mixing of the PC shadow costs with the regime transition probabilities, that repeats :

$$
\begin{aligned}
&p^h \mu_t^{\pi^h} + (1-p^\ell)\,\mu_t^{\pi^\ell} \\
&= p^h \Big[-(\widehat{\pi}_t^h - \pi^{*,CB}) + p^h \mu_{t-1}^{\pi^h} + (1-p^\ell)\,\mu_{t-1}^{\pi^\ell} + \frac{p^h}{\sigma\beta}\,\mu_{t-1}^{y^h} + \frac{(1-p^\ell)}{\sigma\beta}\,\mu_{t-1}^{y^\ell} \Big] \\
&+ (1-p^\ell)\Big[-(\widehat{\pi}_t^\ell - k - \pi^{*,CB}) + p^\ell \mu_{t-1}^{\pi^\ell} + (1-p^h)\,\mu_{t-1}^{\pi^h} + \frac{p^\ell}{\sigma\beta}\,\mu_{t-1}^{y^\ell} + \frac{(1-p^h)}{\sigma\beta}\,\mu_{t-1}^{y^h} \Big]
\end{aligned} \tag{70}
$$

Collecting the contemporaneous and the lagged terms we get:

$$
\begin{aligned}
&p^h \Big[-(\widehat{\pi}_t^h - \pi^{*,CB}) + p^h \mu_{t-1}^{\pi^h} + (1-p^\ell)\,\mu_{t-1}^{\pi^\ell} + \frac{p^h}{\sigma\beta}\,\mu_{t-1}^{y^h} + \frac{(1-p^\ell)}{\sigma\beta}\,\mu_{t-1}^{y^\ell} \Big] \\
&\quad + (1-p^\ell)\Big[-(\widehat{\pi}_t^\ell - k - \pi^{*,CB}) + p^\ell \mu_{t-1}^{\pi^\ell} + (1-p^h)\,\mu_{t-1}^{\pi^h} + \frac{p^\ell}{\sigma\beta}\,\mu_{t-1}^{y^\ell} + \frac{(1-p^h)}{\sigma\beta}\,\mu_{t-1}^{y^h} \Big] \\
&= -p^h\Big(\widehat{\pi}_t^h - \pi^{*,CB}\Big) - (1-p^\ell)\Big(\widehat{\pi}_t^\ell - k - \pi^{*,CB}\Big) \\
&\quad + \Big[(p^h)^2 + (1-p^\ell)(1-p^h)\Big]\mu_{t-1}^{\pi^h} \\
&\quad + \Big[p^h(1-p^\ell) + (1-p^\ell)p^\ell\Big]\mu_{t-1}^{\pi^\ell} \\
&\quad + \frac{1}{\sigma\beta}\Big[(p^h)^2 + (1-p^\ell)(1-p^h)\Big]\mu_{t-1}^{y^h} \\
&\quad + \frac{1}{\sigma\beta}\Big[p^h(1-p^\ell) + (1-p^\ell)p^\ell\Big]\mu_{t-1}^{y^\ell}.
\end{aligned} \tag{71}
$$

Thus we can express the combinations as a function of the contemporaneous inflation gaps and lagged shadow costs.

Realizing that recursively substituting in the shadow costs the multipliers are just ”cumulated” combination of past inflation gaps, weighted with the transition probabilities. Which holds under under a general transversality condition regarding finite shadow costs in the initial period when impact of the initial shadow costs vanishes as the roots of the characteristic polynomial lie within the unit circle.

$$
\begin{aligned}
\mu_t^{\pi^h} &= -\Big(\widehat{\pi}_t^h - \pi^{*,CB}\Big) + p^h \mu_{t-1}^{\pi^h} + (1-p^\ell)\,\mu_{t-1}^{\pi^\ell} + \frac{p^h}{\sigma\beta}\,\mu_{t-1}^{y^h} + \frac{(1-p^\ell)}{\sigma\beta}\,\mu_{t-1}^{y^\ell} = \\
&= -\Big(\widehat{\pi}_t^h - \pi^{*,CB}\Big) - p^h\Big(\widehat{\pi}_{t-1}^h - \pi^{*,CB}\Big) - (1-p^\ell)\Big(\widehat{\pi}_{t-1}^\ell - k - \pi^{*,CB}\Big) + [...] \\
&\quad + \frac{1}{\sigma\beta}\Big(p^h \mu_{t-1}^{y^h} + (1-p^\ell)\,\mu_{t-1}^{y^\ell}\Big)
\end{aligned} \tag{72}
$$

Analogously:

$$\mu_t^{\pi^l} = -\left(\widehat{\pi}_t^\ell - k - \pi^{*,CB}\right) - p^\ell\left(\widehat{\pi}_{t-1}^\ell - k - \pi^{*,CB}\right) - (1-p^h)\left(\widehat{\pi}_{t-1}^h - \pi^{*,CB}\right) + [...]$$
$$+ \frac{1}{\sigma\beta}\left(p^\ell \mu_{t-1}^{y^\ell} + (1-p^h)\,\mu_{t-1}^{y^h}\right) \tag{73}$$

Where the "$\cdots$" indicate additional terms coming from the recursive structure of the shadow costs.

The shadow costs of the IS equations yields:

$$\mu_t^{y^h} = \left\{-\lambda\widehat{y}_t^h + \beta\kappa\left[(\widehat{\pi}_{t+1}^h - \pi^{*,CB}) - p^h\left(\widehat{\pi}_t^h - \pi^{*,CB}\right) - (1-p^\ell)\left(\widehat{\pi}_t^\ell - k - \pi^{*,CB}\right)\right]\right\}$$
$$+ [...] \tag{74}$$

Using that $\mu_t^{y^h} = \mu_t^{y^\ell} = 0, \forall t > 1$, rearranging to write the weighted average of next period's inflation deviations:

$$\frac{\lambda}{\beta\kappa}\widehat{y}_t^h = \left(\widehat{\pi}_{t+1}^h - \pi^{*,CB}\right) - p^h\left(\widehat{\pi}_t^h - \pi^{*,CB}\right) - (1-p^\ell)\left(\widehat{\pi}_t^\ell - k - \pi^{*,CB}\right) + [...] \tag{75}$$

Analogously,

$$\frac{\lambda}{\beta\kappa}\widehat{y}_t^\ell = \left(\widehat{\pi}_{t+1}^\ell - k - \pi^{*,CB}\right) - p^\ell\left(\widehat{\pi}_t^\ell - k - \pi^{*,CB}\right) - (1-p^h)\left(\widehat{\pi}_t^h - \pi^{*,CB}\right) + [...] \tag{76}$$

Turning to the IS curve, the future terms on $\widehat{y}_{t+1}^\ell$ and $\widehat{y}_{t+1}^h$ can be expressed as the weighted function of current and past-period relationships between inflation gaps and the output gaps.

$$
\begin{aligned}
\widehat{y}^h_{t+1} &= \frac{\beta\kappa}{\lambda}\Big[\Big(\widehat{\pi}^h_{t+2}-\pi^{*,CB}\Big)-p^h\Big(\widehat{\pi}^h_{t+1}-\pi^{*,CB}\Big)-(1-p^\ell)\Big(\widehat{\pi}^\ell_{t+1}-k-\pi^{*,CB}\Big)\Big]\\
&\quad+\frac{\beta\kappa}{\lambda}\Big[(p^h)^2+(1-p^\ell)(1-p^h)\Big]\mu_t^{\pi^h}\\
&\quad+\frac{\beta\kappa}{\lambda}\Big[p^h(1-p^\ell)+(1-p^\ell)p^\ell\Big]\mu_t^{\pi^\ell}+[...]\\
&=\frac{\beta\kappa}{\lambda}\Big[\Big(\widehat{\pi}^h_{t+2}-\pi^{*,CB}\Big)-p^h\Big(\widehat{\pi}^h_{t+1}-\pi^{*,CB}\Big)-(1-p^\ell)\Big(\widehat{\pi}^\ell_{t+1}-k-\pi^{*,CB}\Big)\Big]\\
&\quad-\frac{\beta\kappa}{\lambda}\Big[(p^h)^2+(1-p^\ell)(1-p^h)\Big]\Big(\widehat{\pi}^h_t-\pi^{*,CB}\Big)+\\
&\quad-\frac{\beta\kappa}{\lambda}\Big[p^h(1-p^\ell)+(1-p^\ell)p^\ell\Big]\Big(\widehat{\pi}^\ell_t-k-\pi^{*,CB}\Big)+[...] \qquad (77)\\
\widehat{y}^\ell_{t+1} &= \frac{\beta\kappa}{\lambda}\Big[\Big(\widehat{\pi}^\ell_{t+2}-k-\pi^{*,CB}\Big)-p^\ell\Big(\widehat{\pi}^\ell_{t+1}-k-\pi^{*,CB}\Big)-(1-p^h)\Big(\widehat{\pi}^h_{t+1}-\pi^{*,CB}\Big)\Big]\\
&\quad-\frac{\beta\kappa}{\lambda}\Big[(p^\ell)^2+(1-p^h)(1-p^\ell)\Big]\Big(\widehat{\pi}^\ell_t-k-\pi^{*,CB}\Big)+\\
&\quad-\frac{\beta\kappa}{\lambda}\Big[p^\ell(1-p^h)+(1-p^h)p^h\Big]\Big(\widehat{\pi}^h_t-\pi^{*,CB}\Big)+[...] \qquad (78)
\end{aligned}
$$

Next, substitute this expression into the IS equations and rearranging for the policy rate we get:

$$
\begin{aligned}
i_t &= r^n_t+p^h\widehat{\pi}^h_{t+1}+(1-p^h)\widehat{\pi}^\ell_{t+1}+\frac{\beta\kappa\sigma}{\lambda}\Big[p^h\Big(\widehat{\pi}^h_{t+2}-\pi^{*,CB}\Big)+(1-p^h)\Big(\widehat{\pi}^\ell_{t+2}-k-\pi^{*,CB}\Big)\Big]\\
&-\frac{\beta\kappa\sigma}{\lambda}\Big\{\Big[(p^h)^2+(1-p^h)^2\Big]\Big(\widehat{\pi}^h_{t+1}-\pi^{*,CB}\Big)+\Big[p^h(1-p^\ell)+(1-p^h)p^\ell\Big]\Big(\widehat{\pi}^\ell_{t+1}-k-\pi^{*,CB}\Big)\Big\}\\
&-\frac{\beta\kappa\sigma}{\lambda}\Big\{\Big[(p^h)^3+p^h(1-p^\ell)(1-p^h)+(1-p^h)^2p^\ell+(1-p^h)^2p^h\Big]\Big(\widehat{\pi}^h_t-\pi^{*,CB}\Big)\Big\}\\
&-\frac{\beta\kappa\sigma}{\lambda}\Big\{\Big[(p^h)^2(p^\ell)+p^h(1-p^\ell)p^\ell+(1-p^h)p^{\ell 2}+(1-p^h)^2(1-p^\ell)\Big]\Big(\widehat{\pi}^\ell_t-k-\pi^{*,CB}\Big)\Big\}\\
&+\sigma g_t \qquad (79)
\end{aligned}
$$

Simplifying the equation results in

$$
\begin{aligned}
i_t = r_t^n &+ \pi^{*,CB} + (1-p^h)k + \sigma g_t \\
&+ \left[p^h - \frac{\beta\kappa\sigma}{\lambda}A\right] E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right] \\
&+ \left[(1-p^h) - \frac{\beta\kappa\sigma}{\lambda}B\right] E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right] \\
&+ \frac{\beta\kappa\sigma}{\lambda}\Bigg\{ p^h\,\Delta\widehat{\pi}_{t+1}^{HH,h} + (1-p^h)\,\Delta\widehat{\pi}_{t+1}^{LL,\ell} \\
&\qquad + C\,\Delta\widehat{\pi}_t^{HH,h} + D\,\Delta\widehat{\pi}_t^{LL,\ell}\Bigg\},
\end{aligned}
$$

with

$$
\widehat{\pi}_t^{HH,h} = \widehat{\pi}_t^h - \pi^{*,CB}, \qquad \widehat{\pi}_t^{LL,\ell} = \widehat{\pi}_t^\ell - k - \pi^{*,CB}
$$

$$
\Delta\widehat{\pi}_{t+1}^{HH,h} = E_t\left[\widehat{\pi}_{t+2}^{HH,h}\right] - E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right], \quad \Delta\widehat{\pi}_{t+1}^{LL,\ell} = E_t\left[\widehat{\pi}_{t+2}^{LL,\ell}\right] - E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right],
$$

$$
\Delta\widehat{\pi}_t^{HH,h} = E_t\left[\widehat{\pi}_{t+1}^{HH,h}\right] - E_t\left[\widehat{\pi}_t^{HH,h}\right], \quad \Delta\widehat{\pi}_t^{LL,\ell} = E_t\left[\widehat{\pi}_{t+1}^{LL,\ell}\right] - E_t\left[\widehat{\pi}_t^{LL,\ell}\right],
$$

and the composite co-efficients:

$$
\begin{aligned}
A &= (p^h)^2 + (1-p^h)^2, \\
B &= p^h(1-p^\ell) + (1-p^h)p^\ell, \\
C &= (p^h)^3 + p^h(1-p^\ell)(1-p^h) + (1-p^h)^2 p^\ell + (1-p^h)^2 p^h, \\
D &= (p^h)^2 p^\ell + p^h(1-p^\ell)p^\ell + (1-p^h)(p^\ell)^2 + (1-p^h)^2(1-p^\ell).
\end{aligned}
$$

The the policy instrument in the low–credibility regime, i_t^ℓ, is of the same form. Overall the policy rate will follow a two period ahead smoothing of the natural real rate r_t^n and the baseline target $\pi^{*,CB}$; a weighted average of current inflation-gaps.

This is the optimal Taylor–type rule under commitment when the policymaker takes account of future transitions, but abstracts recursive anticipation effect due to expectations of the policy function, that arise because of the Markov–switching nature of the regimes.

We also provide the optimal policy for the quasi-commitment introduced by Schaumburg and Tambalotti (2007) on authors' Github, where past promises regarding the future evolution of the economy do not bind when regime switch, thus after each switch a new commitment can be made.

Lastly, the implementation of loose commitment following Debortoli et al. (2014) in Rise is also available from the authors on request.

D Appendix: Optimality Conditions with Endogenous Transition Probabilities

This appendix provides a detailed analytical derivation of the optimal monetary policy in the Markov switching 3-equation New Keynesian DSGE model with state dependent, endogenous transition probabilities. In what follows we assume that transition probabilities between high and low credibility regimes are influenced by the variance of inflation deviations from the central bank's target. In particular, the endogenous transition probabilities have a steady state value and deviations from it are driven by the squared deviation of inflation deviation from the central bank's target. We assume that in the high credibility regime, the probability of loosing credibility, i.e. the exogenous transition probability form high credibility to low credibility, is increasing in the variation of inflation around the target. The lower the variation, the less likely the credibility put in question. On the other hand, in the low credibility regime, squared deviations of realized inflation from the target undermine the credibility further, reducing the exogenous chances to gain credibility.

$$p_t^h = p_{ss}^h - \tau\left(\widehat{\pi}_t^h - \pi_t^{*,HH,i} - \pi^{*,CB}\right)^2 = p_{ss}^h + \tau\left(\widehat{\pi}_t^h\right)^2 \tag{80}$$

, where τ is a positive constant parameter and p_t^h is constrained to be in $(0,1)$, and the by assumption, yet without loss of generality, inflation target gap of the households aligns with that of the central bank, $\pi_t^{*,HH,i} - \pi^{*,CB} = 0$, we leave to relax it to future research. We provide the discussion of discretion and invite the reader to consult the authors' Github for analytical discussion of endogenous regimes probability under commitment. Under discretion, the central bank re-optimizes given the state of the economy, and interest rates are determined based on the current regime. Therefore in the high credibility regime the central bank maximizes the utility function (V_t^h) as follows:

$$\max_{\widehat{\pi}_t^h, \widehat{y}_t^h, i_t^h} \quad V_t^h = -\frac{1}{2}(\widehat{\pi}_t^h)^2 - \frac{1}{2}\frac{\kappa}{\theta}(\widehat{y}_t^h)^2 + \beta V^h{}_{t+1}$$
$$+\beta\left(1-p_t^h\right)\left(V_{t+1}^\ell - V_{t+1}^h\right) \tag{81}$$

Where the terms of the objective function has been rearranged to recover the no-switch expression, and the welfare gap in the continuation given switching.

This is subject to the NK PC and IS curves:

$$\widehat{y}_t^h = [p_t^h \widehat{y}_{t+1}^h + (1-p_t^h)\widehat{y}_{t+1}^\ell] + \sigma[p_t^h \widehat{\pi}_{t+1}^h + (1-p_t^h)\widehat{\pi}_{t+1}^\ell - i_t^h + r_t^n], \tag{82}$$
$$\widehat{\pi}_t^h = \kappa \widehat{y}_t^h + \beta[p_t^h \widehat{\pi}_{t+1}^h + (1-p_t^h)\widehat{\pi}_{t+1}^\ell]. \tag{83}$$

Then the first order conditions, denoting the shadow prices of the envelop condition, IS and PC curves with μ_t^V $\mu_t^{\widehat{y}_t^h}$ and $\mu_t^{\widehat{\pi}_t^h}$ one gets:

$$\mu_t^V = \beta - \beta\left(1 - p_{ss}^h - \tau\left(\widehat{\pi}_t^h\right)^2\right) \tag{84}$$
$$\widehat{\pi}_t^h = \mu_t^{\widehat{\pi}_t^h}\beta\left(p_{ss}^h + \tau(\widehat{\pi}_t^h)^2\right) - \beta\left(1 - p_{ss}^h - \tau\left(\widehat{\pi}_t^h\right)^2\right)$$
$$+2\beta\left(\widehat{\pi}_t^h\right)^2 - 2\beta\tau\widehat{\pi}_t^h\left(\widehat{\pi}_t^\ell - \widehat{\pi}_t^h\right)$$
$$+\mu_t^{\widehat{y}_t^h}\left(\sigma - \sigma\left(1 - p_{ss}^h - \tau\left(\widehat{\pi}_t^h\right)^2\right) - 2\tau\widehat{\pi}_t^h\left(\widehat{y}_t^\ell - \widehat{y}_t^h\right) - 2\sigma\tau\widehat{\pi}_t^h\right)\left(\widehat{\pi}_t^\ell - \widehat{\pi}_t^h\right)$$
$$+2\beta\tau\widehat{\pi}_t^h\left(V^h{}_{t+1} - V^\ell{}_{t+1}\right) \tag{85}$$
$$0 = \mu_t^{\widehat{y}_t^h}\left(p_{ss}^h + \tau\left(\widehat{\pi}_t^h\right)^2\right) - \frac{\kappa}{\theta}\widehat{y}_t^h + \mathrm{E}_t\left[\mu_{t+1}^V\left(-\mu_{t+1}^{\widehat{y}_t^h} + \kappa\mu_{t+1}^{\widehat{\pi}_t^h}\right)\right] \tag{86}$$
$$0 = \mathrm{E}_t\left[\mu_{t+1}^V \mu_{t+1}^{\widehat{\pi}_t^h}\right] \tag{87}$$
$$0 = \mathrm{E}_t\left[\mu_{t+1}^V \mu_{t+1}^{\widehat{y}_t^h}\right] \tag{88}$$

The key insight from the envelop condition resurfaces: In comparison to the case of constant switching probabilities, the central bank is responding more aggressively to deviations of inflation from the target. In addition to the standard stabilisation criterion of the central bank, it takes into account that the rate decision influences the probabilities of switching to the alternative regime. While the optimal policy in the case of constant switching probabilities, leads to higher rates and a bias in the resulting output gap, the endogenous switching introduces a more aggressive response to surprises in inflation.

Inflation gap and variation, terms involving $\widehat{\pi}_t^h$ and $(\widehat{\pi}_t^h)^2$, is balancing the discounted anticipated welfare gains from high credibility:

$$2\beta\tau\widehat{\pi}_t^h \left(V^h{}_{t+1} - V^\ell{}_{t+1}\right)$$

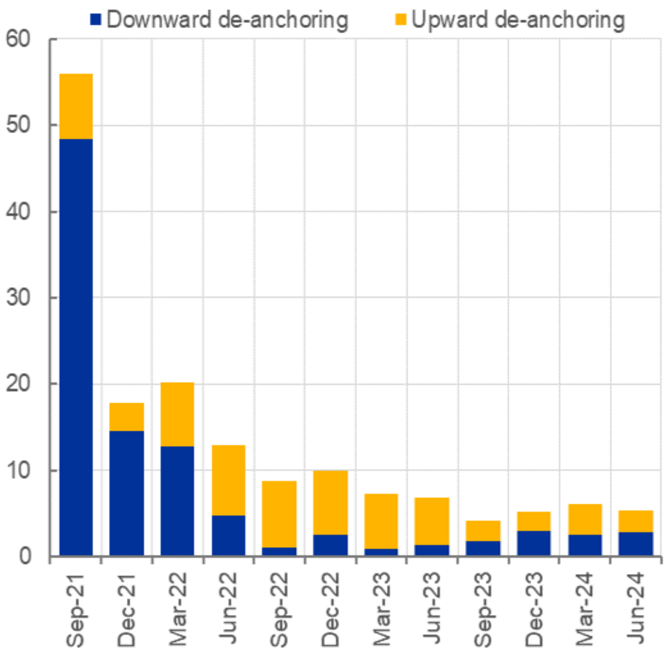
Note that we recover the constant probability case, that is the anticipated welfare gains from high credibility drops out, if either $\tau = 0$ or $\widehat{\pi}_t^h$ is zero, that is if either the transition probability is exogenous, or, the inflation gap is zero, leading to no deviation from target, and thus to no loss of credibility.

The derivation of the low credibility can be found on the authors' Github.

E Appendix: Sensitivities with respect to an alternative calibration of the perceived target

In the main text the calibration of the data generation process of the perceived target is based on the micro-evidence from the Survey of Professional Forecasters discussed in section (1.1). Here we repeat the exercise discussed in section (3.5) assuming instead the calibration proposed in Coenen and Schmidt (2016).

Figure 8: Risks of de-anchoring of medium term inflation expectations
(percentages)

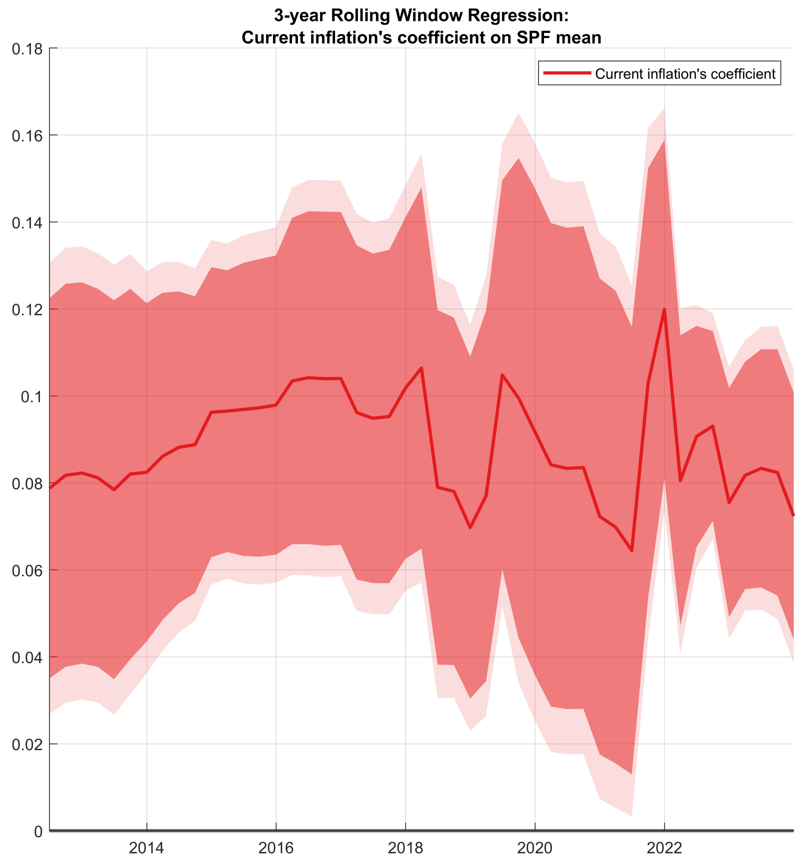


Sources: Authors' calculations.
Notes: The charts show the risk of de-anchoring for the projections from June 2021 to June 2023. The blue bars indicate downward, the red bars indicate upward de-anchoring.
Latest observations: 2024Q2.

Comparing this figure to to figure (4) shows that the

F Appendix: Realized inflation and long-term inflation expectations

Figure 9: Time-varying Relationship between Expected Long-term Inflation and Realized Inflation



Notes: The chart shows the OLS estimates of the coefficient for current inflation on mean SPF long-term inflation expectations from a rolling window sample of 10 years. The shaded areas correspond to 90th and 95th percentiles. *Sources:* SPF, Authors' calculations. Sample: 2002 September - 2023 December.

Figure 9 shows the relationship of the realized inflation with the pooled mean of the long-term inflation expectations. We compute the mean of the pooled SPF responses for each round based on the midpoints of the bins weighted with the associated pooled probabilities and estimate an autoregressive distributed lag model accounting for own lag and current and lagged inflation on a rolling window of 10 years, that is 40 SPF rounds:

$$\bar{\pi}_t^{LR} = \beta_0 + \beta_1 \bar{\pi}_{t-1}^{LR} + \beta_2 \pi_t + \varepsilon_t, \tag{89}$$

where $\bar{\pi}_t^{LR}$ is the mean of the pooled long-term inflation expectations and π_t is the realized HICP for the round conducted. We plot only the realization of current inflation The blue shaded area on Figure 1 shows that there is a significant relationship between long-term inflation expectations and realized inflation, for the 10-year rolling windows ending in mid-2014 up until early 2019. Following the COVID19 pandemic, although the coefficient of contemporaneous inflation increases, the joint effect of current and lagged inflation cancel. Clearly this regression suffers from endogeneity of both SPF expectations and inflation, thus we interpret the evidence as a time-variation of correlation rather than causation. Still, we argue that no such correlation should exist if inflation expectations are firmly anchored.