Endogenous Belief Switching
Revisiting the Forward Guidance Puzzle

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DGMP/MSY

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Disclaimer

The views expressed in this paper are those of the authors, and not necessarily those of the European Central Bank.

FORWARD GUIDANCE AND EXPECTATIONS

Overview

Motivation

"With nominal short-term interest rates at or close to their effective lower bound in many countries, the broader question of how expectations are formed has taken on heightened importance."

Janet Yellen (2016)

- Inflation expectations – become backward looking at the ZLB (Ehrmann, 2015; Carvalho et al., 2019)
- Forward guidance is widely employed at the ZLB
- QE also works through implicit FG, signalling channel

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Problems with Forward Guidance

What is forward guidance?

I consider calendar-based forward guidance: an announcement of setting the short term interest rate to a (low) level for a predetermined horizon.

- Forward Guidance Puzzle: FGP visualized
  - Standard DSGE models tend to overestimate the impact of FG (Del Negro et al., 2012)
  - DSGE models can produce reversals for too long FG (Carlstrom et al., 2015)
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Contributions

Main Contributions

- Explore how forward guidance works in an adaptive learning environment.
- Introduction of endogenous belief switching, that solves and nests the forward guidance puzzle.
- Show how it enables to model endogenous central bank credibility.

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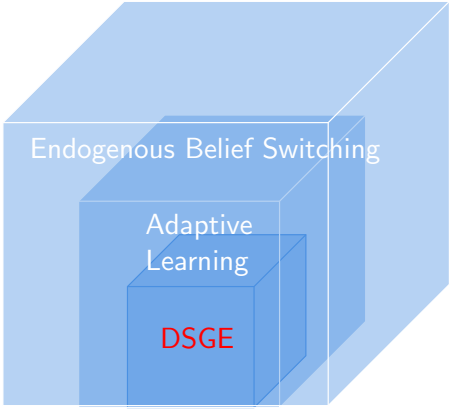
Related Literature

- **Forward guidance and puzzle:** Del Negro et al. (2012); Woodford (2012); Carlstrom et al. (2015); McKay et al. (2016); Caballero and Farhi (2017); Ehrmann et al. (2019)
- **Departures from rational expectations:** Carlstrom et al. (2015); Chung et al. (2015); Maćkowiak and Wiederholt (2009, 2015); Andrade et al. (2019); Molavi (2019)
- **Adaptive learning:** Kydland and Prescott (1982); Marcet and Sargent (1989); Bullard and Mitra (2002); Preston (2005); Evans and Honkapohja (2012); Slobodyan and Wouters (2012); Cole (2015)
- **Regime switching DSGE:** Farmer et al. (2009, 2011); Bianchi (2012); Maih (2015)

Endogenous Belief Switching

- Is a regime switching DSGE.
- Builds on constant gain adaptive learning.
- Beliefs switch following the Switching Kálmán Filter.



Review of RE-DSGE

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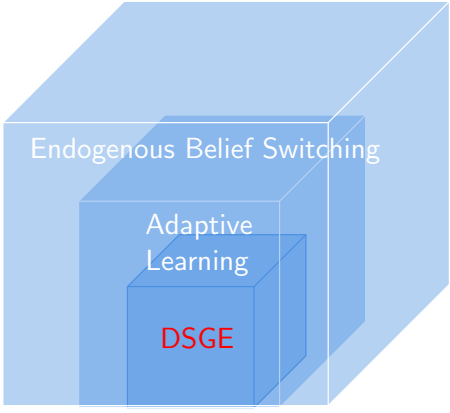

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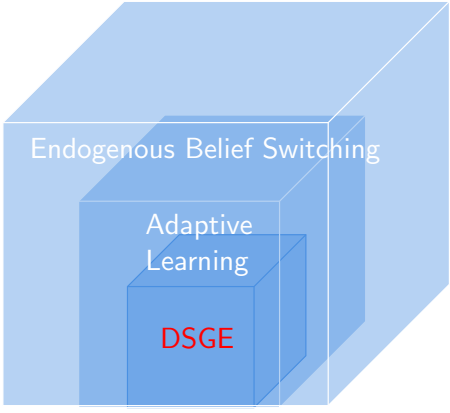

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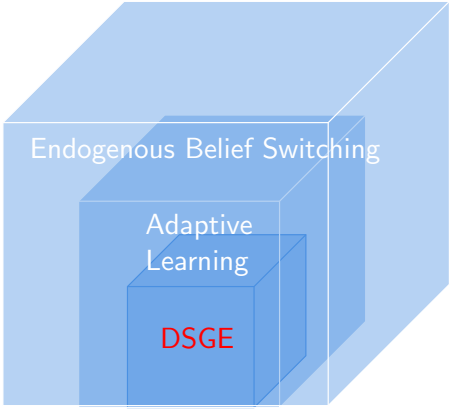

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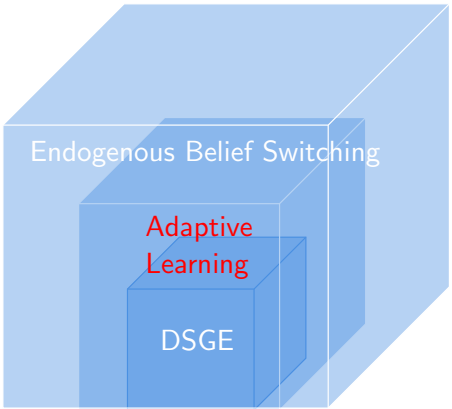

Adaptive Learning

Adaptive expectations

- are **revised**, respond to observed data and learn from forecast errors,
- can be modelled as **forecasting model**,
- can be **backward** and/or **forward** looking.

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DSGE Solution Under Adaptive Learning

A DSGE solution:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t \left[y_{t+1} \right] + B\epsilon_t = const. \tag{1}$$

A DSGE solution given uniqueness:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t, \tag{2}$$

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Replace expectations with Perceived Law of Motion (PLM):

$$E_t \left[y_{t+1} \right] := y_t^f = \alpha_{t-1} + \beta_{t-1}' \begin{bmatrix} y_{t-1} \\ w_t \end{bmatrix} = \Phi_{t-1} \cdot Z_t \tag{3}$$

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The solution becomes the Actual Law of Motion (ALM):

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix}^{ALM} = \mu(\Phi_{t|t-1}, R_{t|t-1}) + T(\Phi_{t|t-1}, R_{t|t-1}) \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_{t|t-1}\epsilon_t. \tag{4}$$

Constant Gain Adaptive Learning - Evolution of Beliefs

Updating linear regression coefficients:

$$\Phi_{t|t} = \Phi_{t|t-1} + \tau R_{t|t}^{-1} Z_t (y_t^{ALM} - y_t^f) \tag{5}$$

Updating mean squared error matrix:

$$R_{t|t} = R_{t|t-1} + \tau (Z_t Z_t' - R_{t|t-1}) \tag{6}$$

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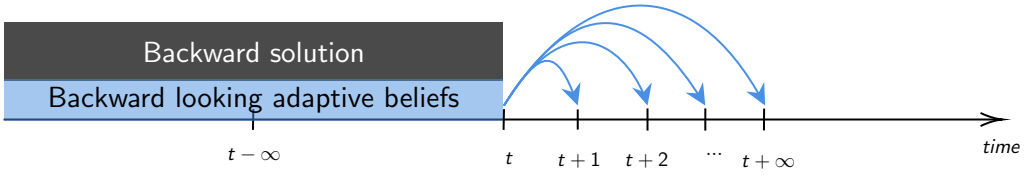
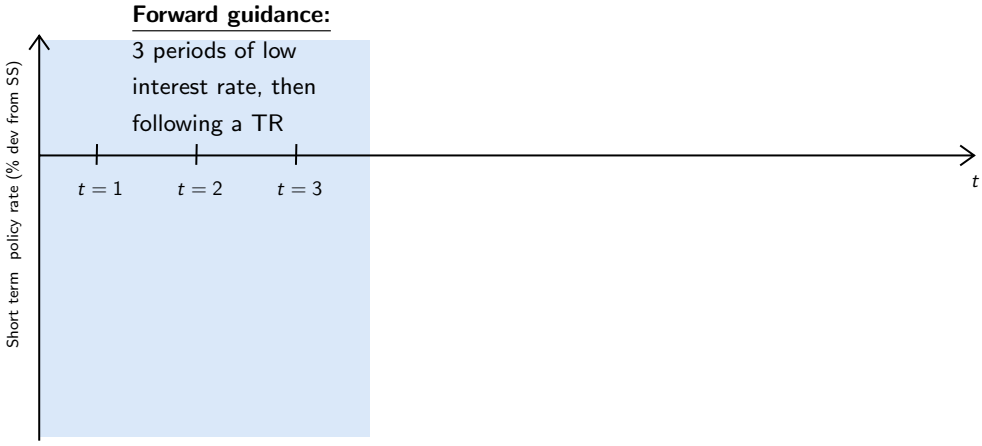
Backward Looking Adaptive Expectations



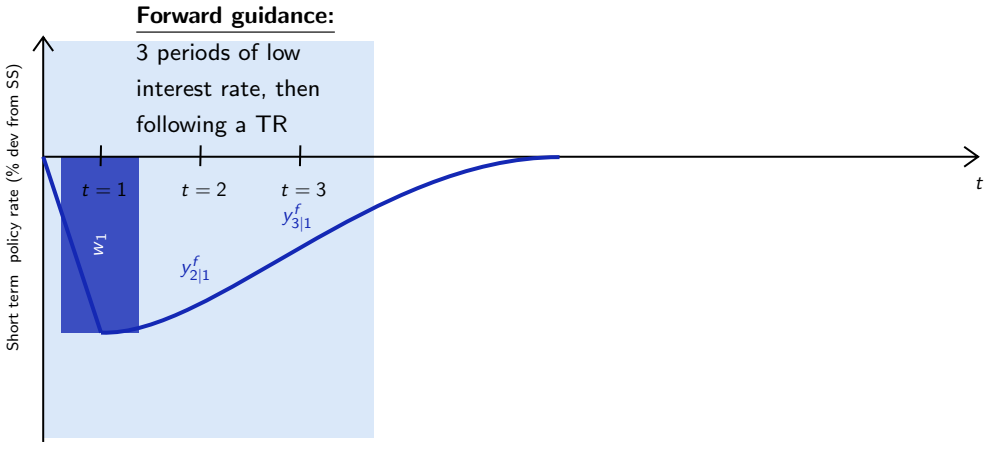
Source: Author's illustration.

Forward Guidance Under Backward Looking Adaptive Expectations: Extended Path Simulation



Source: Author's illustration. (Fair and Taylor, 1983)

Backward looking beliefs perceive it as a pure unanticipated shock, learn it and form expectations ...



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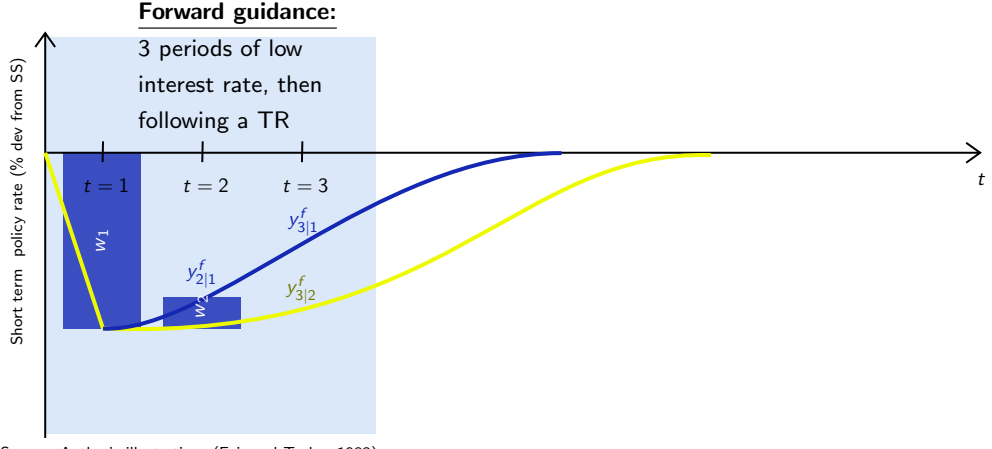
As next period arrives, forward guidance is again seen as a monetary policy shock. The surprise is learnt, making beliefs more persistent.



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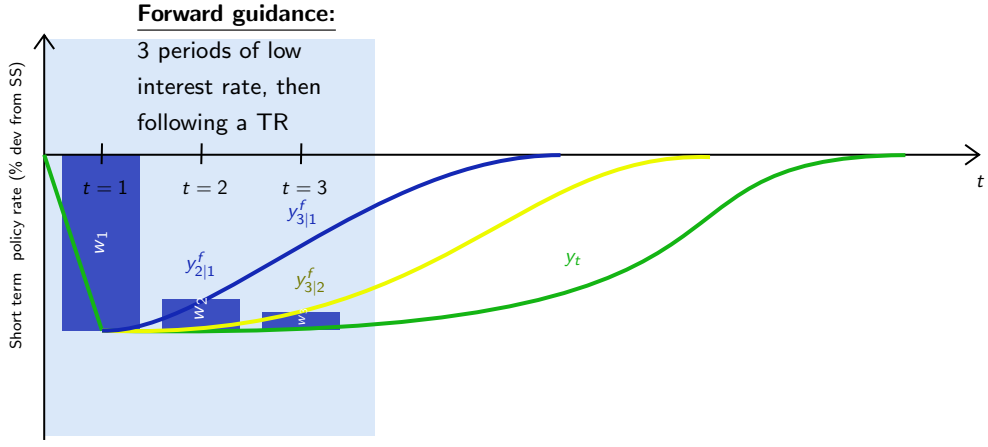
Every consecutive forward guidance period is a smaller monetary policy shock, solving the forward guidance puzzle.



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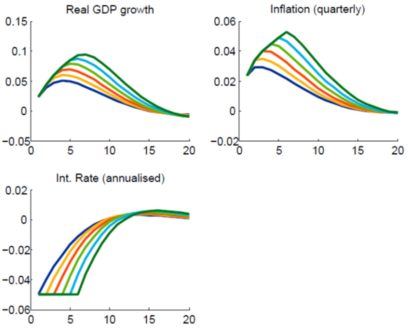
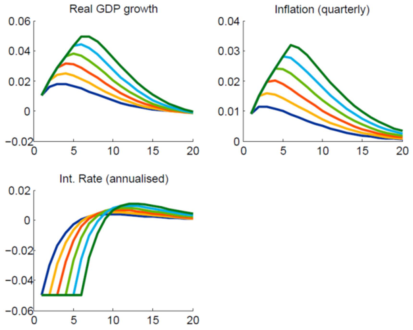
Forward Guidance in the SW07 Model with AL
(slower learning)



Forward Guidance in the SW07 Model with AL
(faster learning)

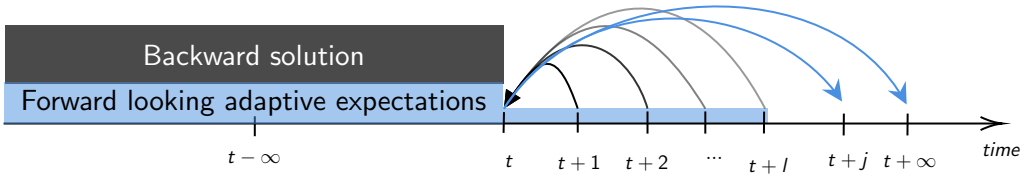


Notes: Forward guidance of setting the interest rate at -0.05 (annualised, quarterly rate) for 1-6 horizon, then following the model's Taylor rule. The blue color represents one horizon, yellow two, red three, green four, light blue five and dark green six periods of low interest rates. The model was solved using the AL tools of the Macromodel Data Base. The model is initiated in the RE SS as in Slobodyan/Wouters (2012). A larger τ means more adaptive expectations and more learning "away" from the RE dynamics. Similarly a lower τ translates to slower learning, less adaptive expectation, agents stick more their RE dynamic beliefs. (Source: Author's calculations)

Forward Looking Adaptive Expectations

$$r_t = \rho r_{t-1} + (1-\rho)(\theta_\pi \pi_t + \theta_x x_t) + \epsilon_t^R + \sum_{l=1}^{L} \epsilon_{t-l}^{R,FG,l} \tag{7}$$

Forward looking beliefs:



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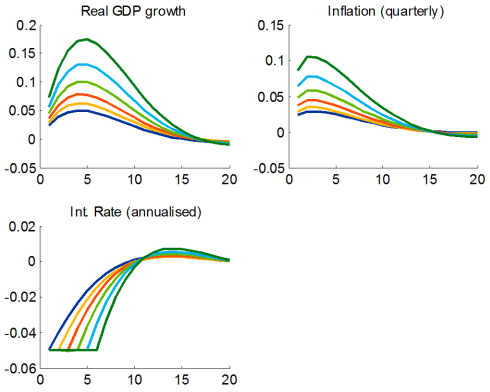

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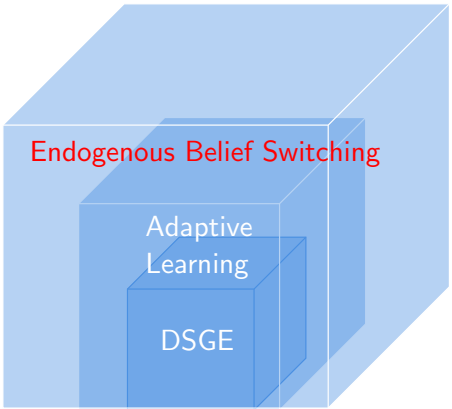


ENDOGENOUS EXPECTATIONS

Endogenous Belief Switching: In SS agents know both forward and backward beliefs, and attach a priori probability to them ...

$Pr(S_t = 1|t-1)$

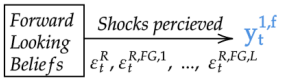
Forward Looking Beliefs

$Pr(S_t = 2|t-1)$

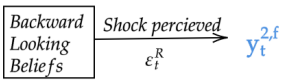
Backward Looking Beliefs

Shocks are perceived and expectations are formed.

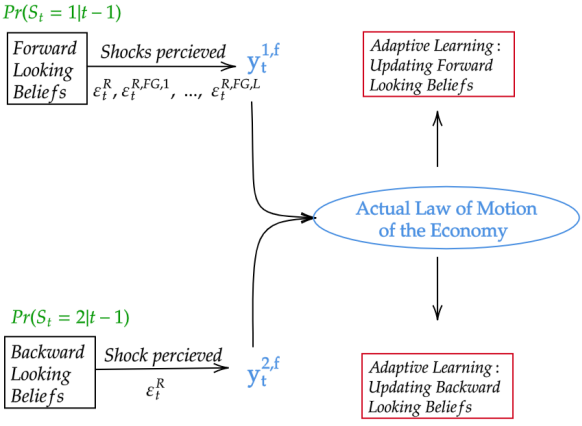
$Pr(S_t = 1|t-1)$



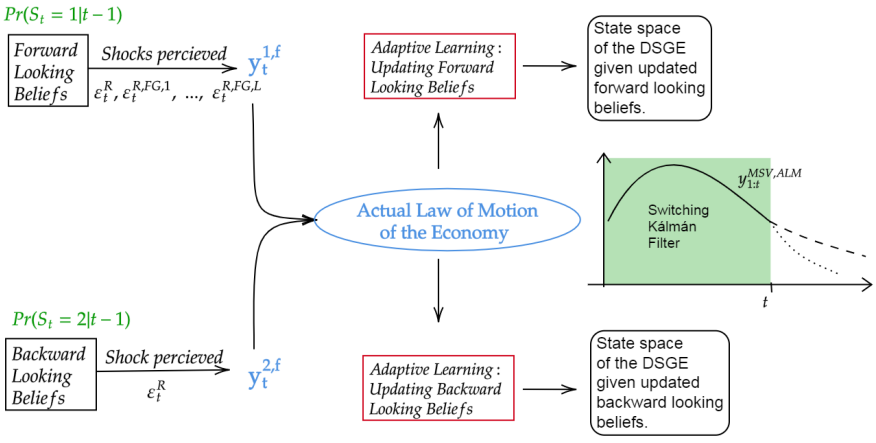
$Pr(S_t = 2|t-1)$



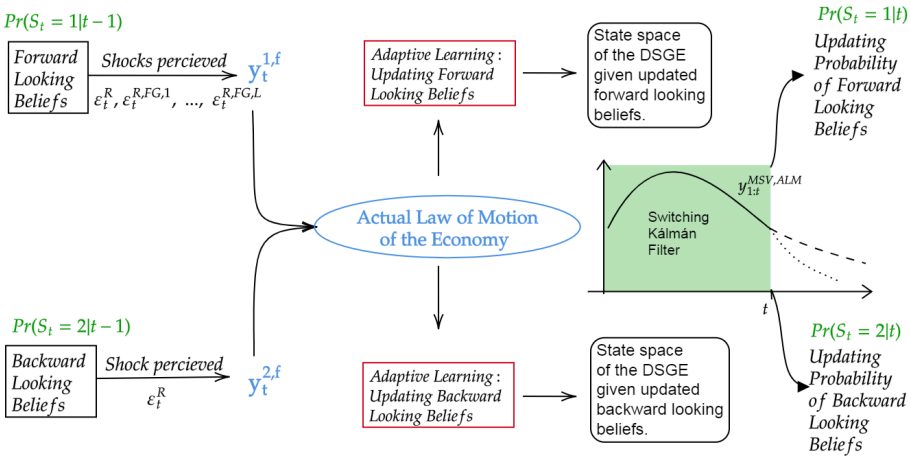
Actual Law of Motion becomes the (a priori) weighted average of the two beliefs. Learning takes place.



Switching Kálmán Filtering of the past: Which belief is the best descriptor of the past and current period?



Outcome of Switching Kálmán Filter: A posterior probability of beliefs being either backward or forward looking. (In Equations)



Three Equation DSGE illustration (Calibration)

1. IS Curve

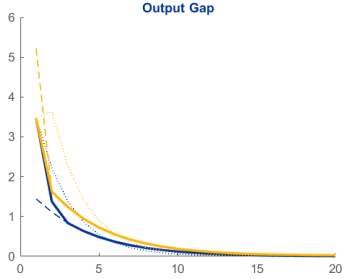
$$x_t = E_t[x_{t+1}] - \frac{1}{\sigma}(r_t - E_t[\pi_{t+1}]) + \epsilon_t^{IS} \tag{8}$$

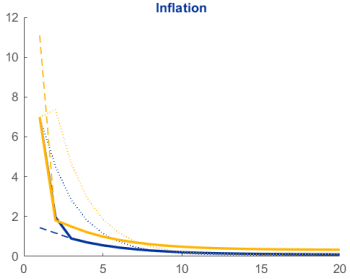
2. Phillips Curve

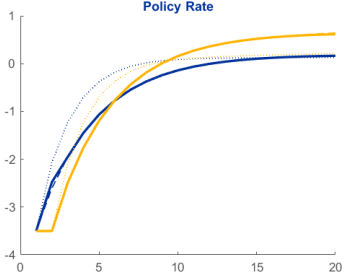
$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(\sigma + \eta)x_t + \kappa r_t \tag{9}$$

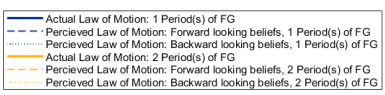
3. Taylor Rule

$$r_t = \rho r_{t-1} + (1-\rho)(\theta_\pi \pi_t + \theta_x x_t) + \epsilon_t^R + \sum_{l=1}^{L} \epsilon_{t-l}^{R,FG,l} \tag{10}$$


Output Gap
6
5
4
3
2
1
0
0
5
10
15
20


Inflation
12
10
8
6
4
2
0
0
5
10
15
20


Policy Rate
1
0
-1
-2
-3
-4
0
5
10
15
20


Actual Law of Motion: 1 Period(s) of FG
Percieved Law of Motion: Forward looking beliefs, 1 Period(s) of FG
Percieved Law of Motion: Backward looking beliefs, 1 Period(s) of FG
Actual Law of Motion: 2 Period(s) of FG
Percieved Law of Motion: Forward looking beliefs, 2 Period(s) of FG
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Three Equation DSGE: Minimum State Variable Solution

Forward looking beliefs

$$\begin{bmatrix} r_t \\ x_t \end{bmatrix} \sim N\left(0, Q_{SS}^{1,RE}\right)$$

$$Q_{SS}^{1,RE} = \begin{bmatrix} 2.1272 & 0.9927 \\ 0.9927 & 0.8580 \end{bmatrix}$$

$$\xrightarrow[\varepsilon_1^R=3.5, \varepsilon_1^{R,FG,2}=3.5]{\text{Adaptive Learning}}$$

$$\begin{bmatrix} r_1 \\ x_1 \end{bmatrix} \sim N\left(0, Q_1^{1,AL}\right)$$

$$Q_1^{1,AL} = \begin{bmatrix} 2.7303 & 0.7438 \\ 0.7438 & 0.9607 \end{bmatrix}$$

Backward looking beliefs

$$\begin{bmatrix} r_t \\ x_t \end{bmatrix} \sim N\left(0, Q_{SS}^{2,RE}\right)$$

$$Q_{SS}^{2,RE} = \begin{bmatrix} 1.1462 & 0.6465 \\ 0.6465 & 0.7318 \end{bmatrix}$$

$$\xrightarrow[\varepsilon_1^R=3.5]{\text{Adaptive Learning}}$$

$$\begin{bmatrix} r_1 \\ x_1 \end{bmatrix} \sim N\left(0, Q_1^{2,AL}\right)$$

$$Q_1^{2,AL} = \begin{bmatrix} 1.3425 & 0.4519 \\ 0.4519 & 0.9247 \end{bmatrix}$$

Three Equation DSGE: Minimum State Variable Solution

Forward looking beliefs

$$\begin{bmatrix} r_t \\ x_t \end{bmatrix} \sim N\left(0, Q_{SS}^{1,RE}\right)$$

$$Q_{SS}^{1,RE} = \begin{bmatrix} 2.1272 & 0.9927 \\ 0.9927 & 0.8580 \end{bmatrix}$$

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$$Q_1^{1,AL} = \begin{bmatrix} 2.7303 & 0.7438 \\ 0.7438 & 0.9607 \end{bmatrix}$$

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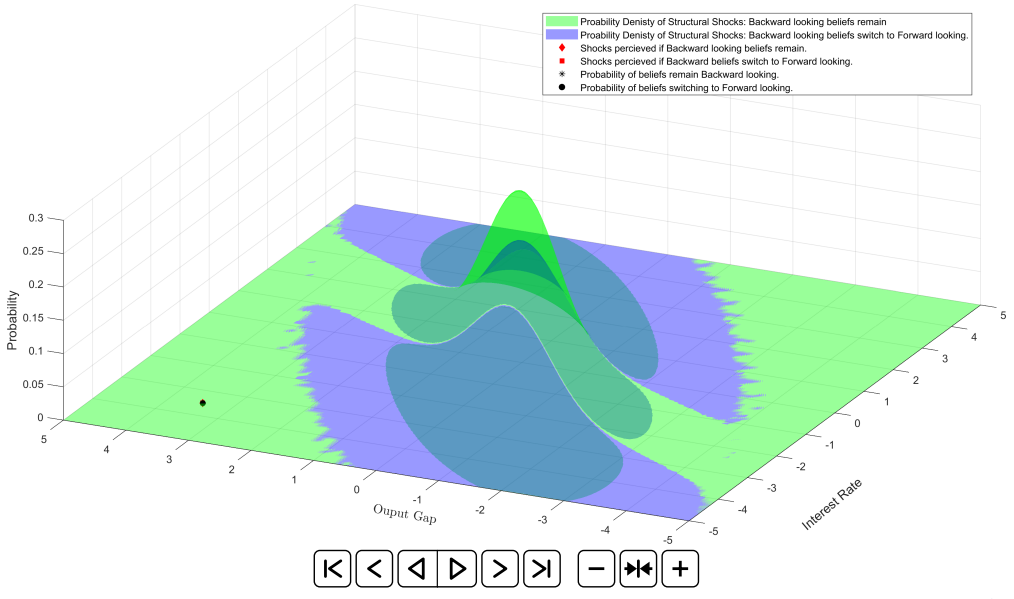
Proability Denisty of Structural Shocks: Backward looking beliefs remain
Proability Denisty of Structural Shocks: Backward looking beliefs switch to Forward looking.
Shocks percieved if Backward looking beliefs remain.
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Probability
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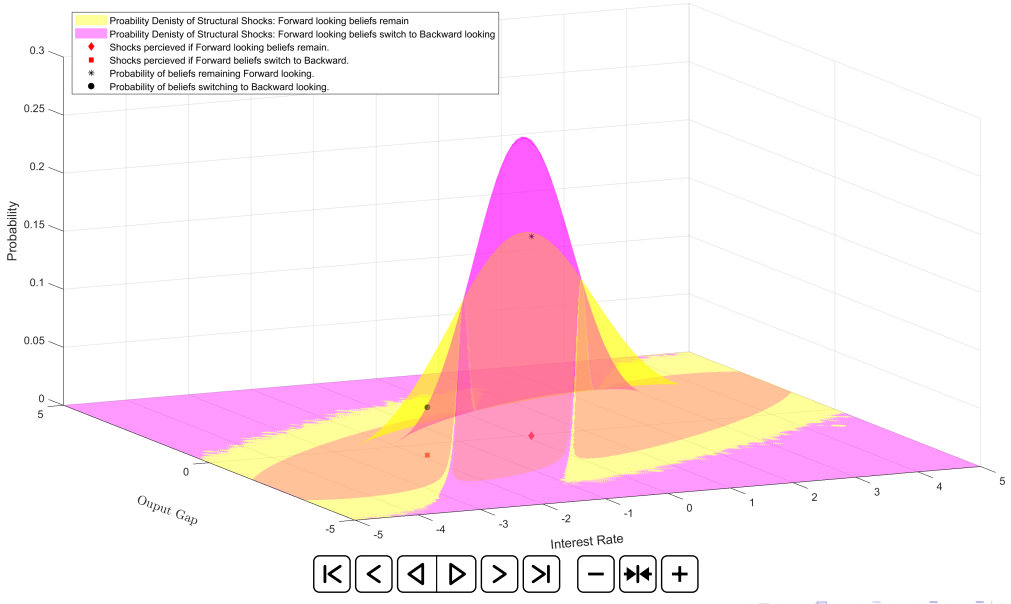
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Interest Rate

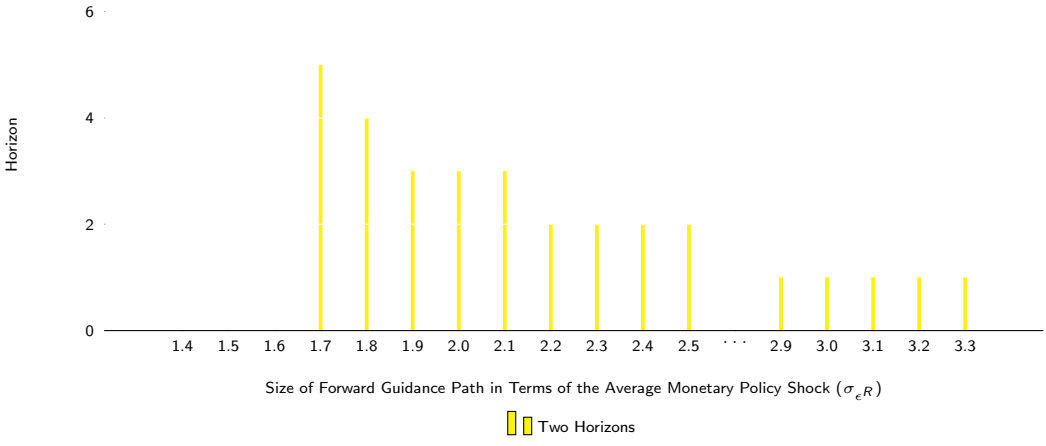
Building Central Bank Credibility Through Forward Guidance

Central bank credibility in Endogenous Belief Switching models

Credibility of forward guidance is gained if agents attach 100% to beliefs being forward looking.

Forward guidance has two dimensions: Size and length.

3EQ: Periods Needed to Switch to Forward Looking Beliefs Due to FG



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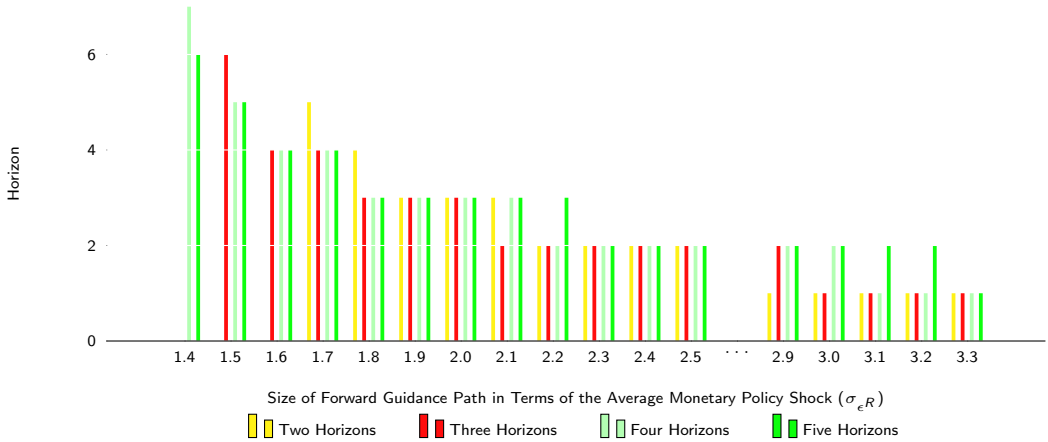

Notes: The chart shows the period when beliefs switch from backward to forward looking as a function of the size and length of the forward guidance. Backward looking beliefs do not respond to future interest rate changes, while forward looking beliefs are of a model with up-to five periods of forward guidance. The size of the path is measured in terms of the standard deviation of the average monetary policy shock. The blue bar indicates one period forward guidance, where the possibility of future forward guidance up to 5 horizons is considered. The yellow, red, light green and green show forward guidance of two, three, four and five horizon respectively. (Source: Author's calculation)

Building Central Bank Credibility Through FG - Results

- A long forward guidance is **hard to believe**, initially...

  at the same time, it **can become credible** as it gets delivered.

- A **large enough forward guidance is always credible**.

- A **small is never**.

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Conclusion

- Constant gain adaptive learning can overcome the forward guidance puzzle if agents are backward looking.
- Endogenous belief switching is a novel framework to study unconventional monetary policy.
- Central banks can gain and lose credibility with forward guidance.

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Policy Implications

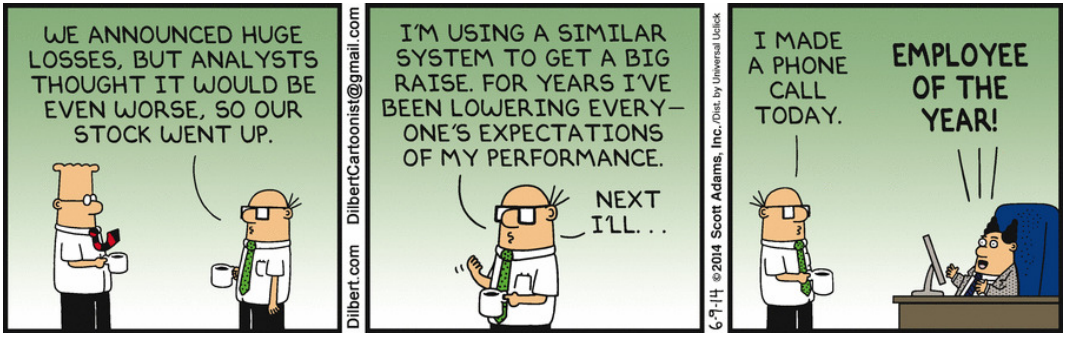
Forward guidance should be bold and persistent to build credibility.

Expectations can become forward looking if the central bank gives an informative, strong signal, either by a large shock or by delivering on past promises.

Endogenous Belief Switching

Endogenous belief switching opens up novel avenues for re-interpretation of the current post-crisis environment.

Thank you for your kind attention!



Source: https://dilbert.com/strip/2014-06-09/

ONE MORE THING ...

Problem of Model Fit - Signal to Noise

State space representation of a DSGE

$$y_t^{state} = \mathbf{F} y_{t-1}^{state} + \mathbf{w}_t \tag{11}$$

$$X_t^{obs} = \mathbf{H} y_t^{state} + \mathbf{u}_t \tag{12}$$

$$TSS = ESS + RSS \tag{13}$$

$$R^2 = 1 - \frac{RSS}{TSS} \tag{14}$$

The accuracy of DSGE filtering determines how much of the movement in states is credited to the central bank.

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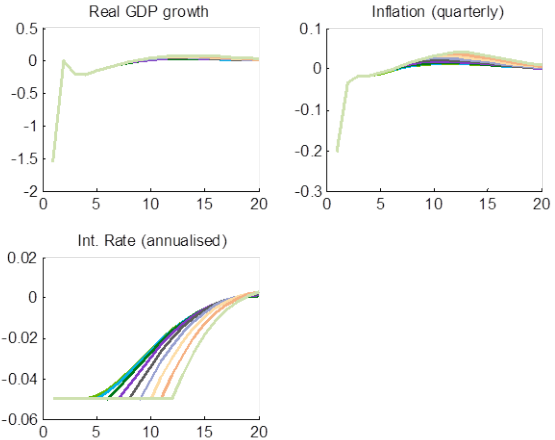
Signal to Noise - Take Away:

- $R^2 \simeq 0$:
  The communicated DSGE has no explanatory power: no information can be gained, no switching takes place, initial beliefs remain.
- $R^2 \simeq 50$:
  Ample uncertainty,i.e. the DSGE is an imperfect filter for the economy: forward guidance will be difficult to implement. Backward looking beliefs become the equilibrium expectations.
- $R^2 \simeq 100$:
  No unexplained uncertainty, i.e. DSGE is a perfect filter for the economy: Credibility is the easiest to gain, and maintain. Presented today.

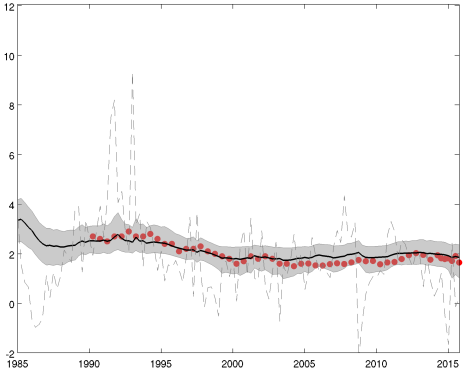
State Contingent Forward Guidance with Endogenous Belief Switching



Notes: Forward guidance of setting the interest rate at -0.05 (annualised, quarterly rate) for 1-12 horizon, then following the model's Taylor rule. The economy is pushed to the path by a sequence of risk premium shock for 4 periods, thus the shadow rate is negative throughout. (Source: Author's calculations)

De-Anchoring of Expectations: Expectations Become Adapitive

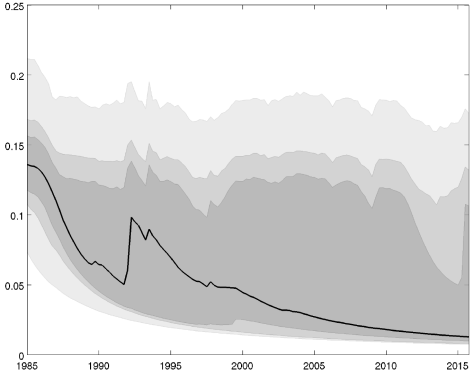
Inflation Expectations in Germany



Notes: Dashed line denotes the HICP, black solid line measures the model estimate for long-term inflation expectations, red dots are the 5-10Y Consensus forecasts.

Source: Carvalho et al. (2019, p. 39), ◂ Other EA countries.

Estimates of the Learning Gain Parameter

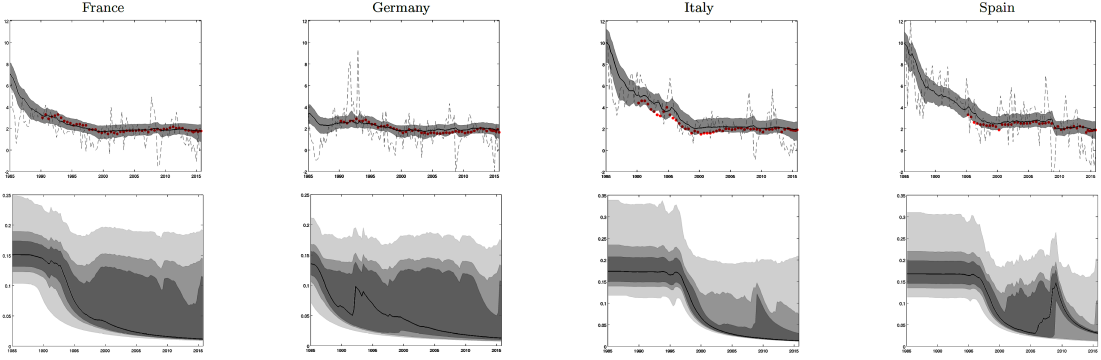


Notes: Black line shows the median, grey area measures the 50th, 70th and 95th credible intervals.

De-anchoring of Expectations: Adaptive Learning Gain Picks Up

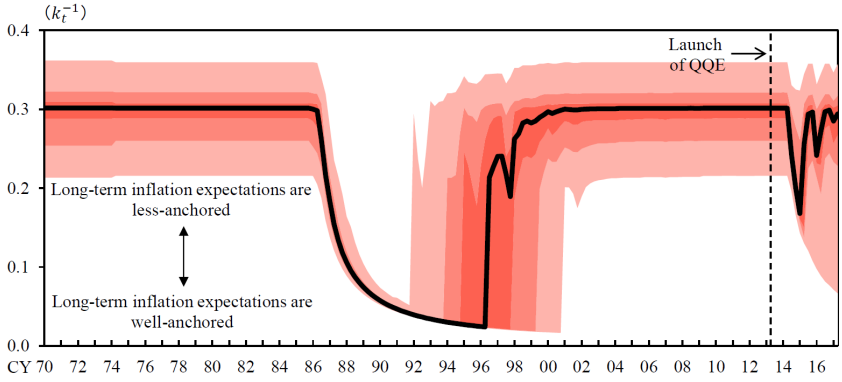
These panel show model predictions for long-term inflation forecasts (top) and the learning gain (bottom). Black solid line denotes the median; the grey areas measure the 50th, 70th and 95th credible intervals; the red dots denoted the five-to-ten inflation forecasts from Consensus Economics.

EMU Countries



Source: Carvalho et al. (2019, p. 39) Back to Germany.

Re-anchoring of Expectations with QE in Japan



Notes: The shaded areas in panel (b) shows the 5-95, 20-80, and 35-65 percentiles of the posterior distribution of the learning gain. The solid line indicates the median of the posterior distribution.
Sources: Ministry of Internal Affairs and Communications; Japan Center for Economic Research; Cabinet Office; Consensus Economics; Bank of Japan.

Source: ?, p. 39

DSGE Solution Under Rational Expectations

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t \left[y_{t+1} \right] + B\epsilon_t = const. \tag{15}$$

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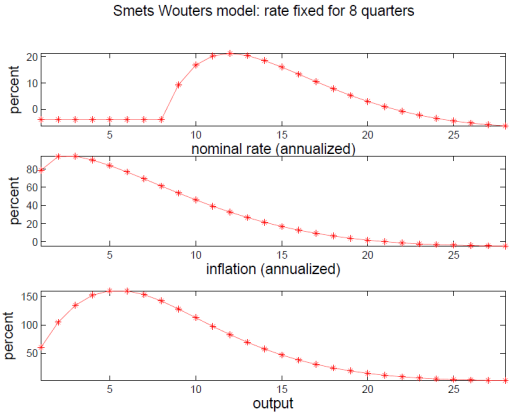
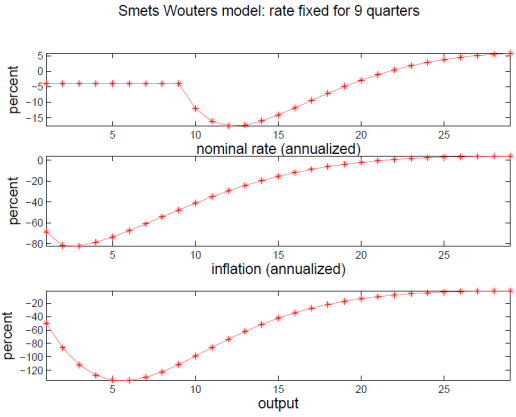
A DSGE solution, given uniqueness:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t, \tag{16}$$

The forward guidance puzzle





Source: Carlstrom et al. (2015)

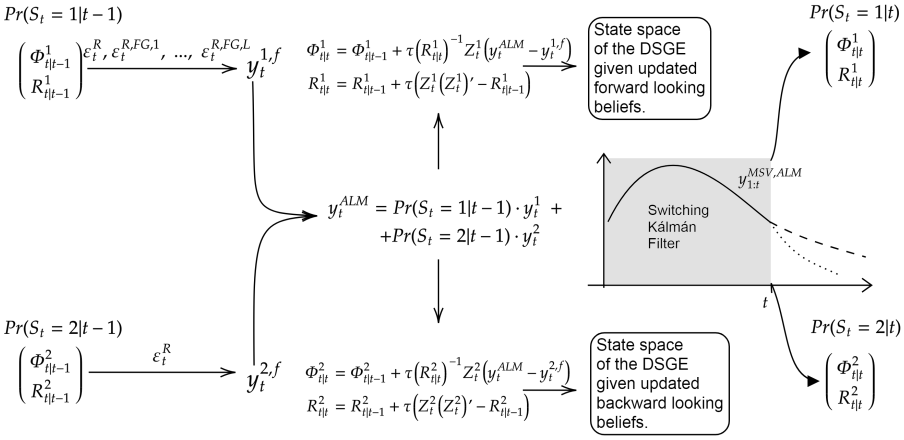
Back to Forward guidance.

Three Equation DSGE Calibration

Parameter	Value	Description
β	0.99	Discount factor
η	1	Frisch elasticity
κ	0.0858	Slope of the PC
ρ	0.900	Interest rate smoothing
θ_π	0.150	Inflation response
θ_x	0.013	Output gap response
$\sigma_{\epsilon^{IS}}$	1	Standard error of IS curve shock
$\sigma_{\epsilon^{R}}$	1	Standard error of Monetary Policy shock
$\sigma_{\epsilon^{R,FG,l}}$	1	Standard error of l period ahead forward guidance shock

Source: Ravenna and Walsh (2006)

Back to Equations.



Back to ◂ Diagram.

Endogenous Beliefs Timing Assumptions

1. At the beginning of period t, the agents inherit the beliefs formed in the previous period.
2. Agents form expectations, based on information set (PLM).
3. The current state is determined as the solution of the DSGE, given beliefs (ALM).
4. Updated beliefs are used to estimate to probability that either regime fits better the observed minimum state variables.

Andrade, P., G. Gaballo, E. Mengus, and B. Mojon (2019, July). Forward guidance and heterogeneous beliefs. *American Economic Journal: Macroeconomics 11*(3), 1–29.

Bhattarai, S., G. B. Eggertsson, and B. Gafarov (2015, July). Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing. NBER Working Papers 21336, National Bureau of Economic Research, Inc.

Bianchi, F. (2012). Regime switches, agents' beliefs, and post-world war ii us macroeconomic dynamics. *Review of Economic studies 80*(2), 463–490.

Bullard, J. and K. Mitra (2002). Learning about monetary policy rules. *Journal of Monetary Economics 49*(6), 1105 – 1129.

Caballero, R. and E. Farhi (2017, 02). The Safety Trap. *The Review of Economic Studies 85*(1), 223–274.

Carlstrom, C. T., T. S. Fuerst, and M. Paustian (2015). Inflation and output in new keynesian models with a transient interest rate peg. *Journal of Monetary Economics 76*, 230 – 243.

Carvalho, C., S. Eusepi, E. Moench, and B. Preston (2019, February). Anchored inflation expectations. Working paper, CEPR.

Chung, H., E. Herbst, and M. T. Kiley (2015). Effective monetary policy strategies in new keynesian models: A reexamination. *NBER Macroeconomics Annual 29*(1), 289–344.

Cole, S. (2015). Learning and the effectiveness of central bank forward guidance. Technical report, University Library of Munich, Germany.

Del Negro, M., M. P. Giannoni, and C. Patterson (2012). The forward guidance puzzle. *FRB of New York Staff Report* (574).

Eggertsson, G. (2006). The deflation bias and committing to being irresponsible. *Journal of Money, Credit and Banking 38*(2), 283–321.

Ehrmann, M. (2015). Targeting inflation from below: How do inflation expectations behave? *International Journal of Central Banking*.

Ehrmann, M., G. Gaballo, P. Hoffmann, and G. Strasser (2019). How to signal the future path of interest rates? The international evidence on forward guidance. *Research Bulletin 61*.

Eusepi, S. and B. Preston (2011). Expectations, learning, and business cycle fluctuations. *The American Economic Review 101*(6), 2844–2872.

Evans, G. W. and S. Honkapohja (2012). *Learning and expectations in macroeconomics*. Princeton University Press.

Fair, R. C. and J. B. Taylor (1983). Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models. *Econometrica 51*(4), 1169–1185.

Farmer, R. E., D. F. Waggoner, and T. Zha (2009). Understanding markov-switching rational expectations models. *Journal of Economic theory 144*(5), 1849–1867.

Farmer, R. E., D. F. Waggoner, and T. Zha (2011). Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamics and Control 35*(12), 2150 – 2166. Frontiers in Structural Macroeconomic Modeling.

Kydland, F. and E. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*(6), 1345–70.

Maih, J. (2015). Efficient perturbation methods for solving regime-switching dsge models. *Norges Bank Working Paper 1— 2015*.

Marcet, A. and T. J. Sargent (1989). Convergence of least squares learning mechanisms in self-referential linear stochastic models. *Journal of Economic theory 48*(2), 337–368.

Maćkowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review 99*(3), 769–803.

Maćkowiak, B. and M. Wiederholt (2015, 08). Business Cycle Dynamics under Rational Inattention. *The Review of Economic Studies 82*(4), 1502–1532.

McKay, A., E. Nakamura, and J. Steinsson (2016). The power of forward guidance revisited. *American Economic Review 106*(10), 3133–58.

Molavi, P. (2019). Macroeconomics with learning and misspecification: A general theory and applications. MIT, Job Market Paper, available at: `https://economics.mit.edu/files/16326`.

Preston, B. (2005). Learning about monetary policy rules when long-horizon expectations matter. *International Journal of Central Banking 1*(2).

Ravenna, F. and C. E. Walsh (2006). Optimal monetary policy with the cost channel. *Journal of Monetary Economics 53*(2), 199 – 216.

Slobodyan, S. and R. Wouters (2012). Learning in an estimated medium-scale dsge model. *Journal of Economic Dynamics and control 36*(1), 26–46.

Woodford, M. (2012). Methods of policy accommodation at the interest-rate lower bound. The Changing Policy Landscape: 2012 Jackson Hole symposium. Federal Reserve Bank of Kansas City.